Unambiguous trends combining absolute and relative income poverty: new results and global application.*

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Abstract

Over 1990-2015, many countries experienced a reduction in extreme absolute poverty and an increase in relative poverty. As a result, the global trend of "overall" income poverty, which combines absolute and relative poverty, may depend on arbitrary normative choices such as the priority given to the absolutely poor over the relatively poor. We prove that, if we assume that an individual who is absolutely poor is poorer than an individual who is only relatively poor, the overall poverty trend is sometimes independent of the priority parameter, even for cases for which absolute and relative poverty follow opposite trends. We conduct a survey that suggests that this normative assumption collects broad support. We apply overall poverty measures satisfying this assumption to assess the evolution of global poverty over 1990-2015. We find that global overall poverty has been (at least) halved, regardless of the value chosen for the priority parameter.

JEL: D63, I32.

Keywords: Income Poverty, Relative Poverty, Absolute Poverty, Global Poverty.

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1 Introduction

The first and maybe the most prominent of the Millennium Development Goals was to halve extreme income poverty by 2015, taking 1990 as reference year. An individual is considered extremely poor if her income is lower than \$1.9 per day in 2011 Purchasing Power Parity (PPP) (Ferreira et al., 2016). This absolute poverty threshold reflects the poverty standards in the poorest countries, which typically capture the minimal resources necessary for nutrition and other *subsistence* needs. By 2015, this goal had been reached by a large margin (World Bank, 2018). Over the same period, besides a strong growth, many countries experienced an increase in within-country income inequality (Bourguignon, 2015; Milanovic, 2016). In such countries, the rising inequality may have translated into greater *social exclusion* of the poor because relatively deprived individuals experience difficulties to engage in the everyday life of their society (Townsend, 1979; Ravallion, 2008). This evolution is captured by rising relative poverty indicators. If absolute poverty decreases while relative poverty increases, one may wonder how global income poverty has evolved over the period 1990-2015 when adopting a definition of income poverty that accounts for both its absolute and relative aspects.

There are several reasons for adopting an "overall" definition of poverty, which conflates absolute and relative poverty. First, such definition is necessary when taking a world approach to economic poverty and exclusion. As argued by Atkinson and Bourguignon (2001), taking a world approach requires a framework that unifies the poverty measurement practices in developing countries and in developed countries. If the former rely on absolute measures typically grounded in subsistence, the latter use relative measures grounded in social participation. Second, such definition is also necessary when being poor is interpreted as having insufficient welfare and individual welfare depends on both own income and relative income (Ravallion, 2008; Ravallion and Chen, 2011). There is now ample evidence that relative income is an important determinant of subjective well-being (Clark and Oswald, 1996; Luttmer, 2005; Perez-Truglia, 2020). More generally, if subsistence and social participation are two determinants of welfare, then welfare depends on own income and relative income. Finally, such definition is necessary when policy makers that care for both subsistence and social inclusion, such as the World Bank (2015), need to select among policies that affect both absolute and relative poverty.[1]

When absolute and relative measures show opposite trends, the direction of overall poverty change may depend on some parameters entering its definition. Most notably, the evolution of overall poverty may depend on the normative weight that captures the *priority* assigned to the absolutely poor. This priority measures how much more (or less) overall poverty is reduced when an additional unit of income is given to an absolutely poor

[1] Some pro-growth policies may generate larger inequalities while some redistribution policies may have disincentive effects that may hinder growth.

individual rather than to an individual who is only relatively poor. Also, the evolution of overall poverty may depend on the exact definition of the poverty lines. Indeed, adopting a more "demanding" relative line mechanically increases the importance given to social participation in the overall measure. This dependence on such arbitrarily chosen parameters significantly limits the usefulness of overall poverty measures.

In this paper, we show under a rather mild normative assumption that global *overall* income poverty has been (at least) halved over 1990-2015, *regardless of the value chosen for the priority parameter*. Moreover, this result is highly robust and in particular holds for alternative specifications of the relative line. The magnitude of poverty reduction is much larger than what alternative overall poverty measures find. To reach this result, we make use of a recently proposed family of overall poverty indices parametrized by the priority given to the absolutely poor (Decerf, 2017). We provide a new result for this family of indices: all indices may agree on the direction of overall poverty change even in some cases for which absolute and relative measures show opposite trends. This result follows from the fact that these indices satisfy our normative assumption.

Our analysis relies on a normative assumption stating that an individual who is absolutely poor is *poorer* than an individual who is only relatively poor, regardless of the income standard in their respective societies. That is, an absolutely poor individual in a low-income country cannot be considered less poor than a relatively poor individual in a higher income country whose personal income is above the absolute threshold. Under a world approach, this assumption amounts to granting a form of precedence to subsistence over social participation. Atkinson and Bourguignon (2001), and later Decerf (2017), express support for granting some form of precedence to subsistence over social participation.[2] Under a welfarist approach, this assumption implies that individual welfare depends on relative income, but only above the absolute threshold. This corresponds to a Maslovian view whereby individuals give precedence to their subsistence needs, at least when these needs are not minimally satisfied. We conduct an online survey in the US, the UK and South Africa in order to test whether our assumption collects considerable support under both of its interpretations. In all three countries, the vast majority of respondents state that (1) an individual deprived in terms of subsistence needs is poorer than an individual deprived in terms of social participation and (2) they would themselves prefer to be deprived in terms of social participation rather than deprived in terms of subsistence.[3]

Our normative assumption plays a key role in the possibility to make overall poverty comparisons that are independent of the value chosen for the priority parameter. We

[2]Alternatively, Decerf (2021) recently shows that, in the presence of two poverty lines, an overall poverty measure satisfies a set of basic axioms a la Foster and Shorrocks (1991) only if it meets our normative assumption.

[3]These results are in line with a similar questionnaire experiment conducted over a population of university students in several countries by Corazzini et al. (2011).

illustrate this based on an example for which absolute and relative poverty follow opposite trends. Consider an income distribution for which the absolute poverty threshold is lower than the relative poverty threshold. Assume that this distribution has only one poor individual and this individual is absolutely poor. Consider a second distribution that is obtained from the first distribution by a particular form of unequal growth: the income of all individuals increases, the income of the poor individual is lifted above the absolute threshold, but her income increases at a slower pace than the relative threshold. The poor individual is only relatively poor in the second distribution but her income is now further away from the relative threshold. Therefore, relative poverty is larger in the second distribution. Our normative assumption implies that overall poverty is unambiguously larger in the first distribution because the only poor individual is absolutely poor in the first but not in the second distribution.

We characterize the conditions under which overall poverty comparisons are independent of the priority assigned to an absolutely poor over an only relatively poor. In the family of indices considered, the two extreme values of this parameter attribute zero and infinite priority to the absolutely poor, respectively. The necessary and sufficient conditions obtained are easy to use. The reason is that all family members yield poverty comparisons that lie between those yielded by the two extreme family members. Therefore, these conditions allow us to place a lower and upper bound on the extent of overall poverty reduction.

Using World Bank data, we show empirically that when we measure global poverty all indices in the family considered have declined by at least 50% from 1990-2015. The extent of overall poverty reduction is considerably large. This result is not entirely driven by the tremendous progress achieved by one or two populous countries such as China or India. In fact, our result holds for almost one fourth of all countries, when taken individually. Our result is robust to five different definitions of the relative line. These alternative specifications reflect to a large extent the variety of proposals made in the literature (Atkinson and Bourguignon, 2001; Chen and Ravallion, 2013; Jolliffe and Prydz, 2016; World Bank, 2018). In particular, we consider mean as well as median-sensitive relative poverty lines and we allow for different values for their slope and intercept. Our finding confirms and strengthens positive evaluations of the success achieved against global income poverty over 1990-2015.

Alternative measures find much less overall poverty reduction than our lower bound estimate. The reason is that these alternative measures behave as *relative* measures as soon as the relative threshold is larger than the absolute threshold, *i.e.* as soon as the income standard reaches a certain value. Beyond that point, alternative measures violate our normative assumption and therefore need not record any progress when economic growth lifts individuals out of absolute poverty. For the set of countries whose relative threshold is larger than the absolute threshold, we show that all our measures find a rate of poverty reduction that is on average several times higher than the one found using the most well-known alternative measure. For some cases, such as urban China from 1996, the alternative measure increases while the entire family of measures that we consider decline.

Our empirical analysis has an additional implication for the literature on global poverty measurement. Our results further suggest that the selection of poverty indices, which has been largely neglected by the literature, affects the magnitude of overall poverty reduction at least as much as the selection of poverty lines. Specifically, similar differences in trends emerge when we compare our results with those obtained from standard indices to measuring overall poverty than when we change the lines keeping the index constant.

The rest of the paper is organized as follows: we present the theory in Section 2, the survey design and results in Section 3, the data and our main specification in Section 4, the empirical analysis in Section 5, and we conclude in Section 6.

2 Absolute, relative and overall poverty measures

In this theoretical section, we present the basic framework, introduce the four families of poverty measures that we use, and study the conditions under which the family of overall measures that satisfy our assumption yields unambiguous comparisons.

2.1 Basic Framework

Let an income distribution $y := (y_1, \ldots, y_n)$ be a list of non-negative incomes sorted in non-decreasing order, with $n \in \mathbb{N}$. The set of such income distributions is denoted by Y. Let $\overline{y}$ denote the income standard in distribution y, *e.g.* mean or median income in y. The income standard is homogeneous of degree one. We consider two different poverty status, each identified by a specific poverty line.

The *absolute* poverty line is defined by a poverty threshold $z_a \in \mathbb{R}_{++}$, which does not depend on the income standard. An individual i is deemed absolutely poor if $y_i < z_a$. We interpret z_a to be the minimal income level allowing to purchase the goods necessary to satisfy basic needs (*e.g.* food, clothes or shelter). The number of absolutely poor individuals in distribution y is denoted by $q_a(y)$.

The *relative* poverty line is defined by a threshold function $z_r : \mathbb{R}_+ \to \mathbb{R}_+$ defined as $z_r(\overline{y}) = b + s\overline{y}$, where $s \in (0, 1)$ is the slope of the relative line and $b \geq 0$ is its intercept. Strongly relative poverty lines have $b = 0$ and weakly relative lines have $b > 0$ (Ravallion and Chen, 2011). Typically, the slope takes value $s = 0.5$. An individual i is deemed relatively poor if $y_i < z_r(\overline{y})$. We interpreter the relative threshold $z_r(\overline{y})$ as the minimal amount necessary to engage in the everyday life of a society whose income standard is $\overline{y}$. The number of relatively poor individuals in distribution y is denoted by $q_r(y)$.

A poverty measure is a function $P : Y \to [0,1]$ that ranks all income distributions using a fixed (set of) poverty line(s). If for two distributions $x, y \in Y$ we have $P(x) > P(y)$, then x has more poverty than y. We say that P measures *absolute* (resp. *relative*) poverty if P identifies the poor using only the absolute (resp. relative) line. We say that P measures *overall* poverty if P identifies the poor using both the absolute and the relative line. In this latter case, the number of individuals who are poor is denoted by $q(y) = max\{q_a(y), q_r(y)\}$ and the number of individuals who are only relatively poor is $q(y) - q_a(y)$. Since income distributions are sorted, if $i \leq q_a(y)$ then individual i is absolutely poor and if $q_a(y) + 1 \leq i \leq q(y)$ then individual i is only relatively poor.

2.2 Four families of additive poverty measures

We consider four families of additive poverty measures: one family of absolute measures, one family of relative measures and two families of overall measures, which combine absolute and relative poverty. Only one of the two families of overall measures satisfies our normative assumption. All measures considered are additive, which implies that poverty is measured as the average poverty contribution of all individuals in a distribution.

First, the absolute family is based on the Foster-Greer-Thorbecke (FGT) indices (Foster et al., 1984). The *absolute* FGT poverty measures A_α are defined as:

$$A_\alpha(y) \coloneqq \frac{1}{n}\sum_{i=1}^{q_a(y)}(1 - d_a(y_i))^\alpha \qquad \text{where} \qquad d_a(y_i) = \frac{y_i}{z_a}, \tag{1}$$

where function d_a computes the *normalized income*, *i.e.* the income divided by the poverty threshold, and the poverty aversion parameter $\alpha \geq 0$ tunes the priority given to poor individuals with smaller normalized income. This family admits the head-count ratio ($\alpha = 0$) and the poverty-gap ratio ($\alpha = 1$) as special cases.

Second, the *relative* FGT poverty measures R_α are defined as:

$$R_\alpha(y) \coloneqq \frac{1}{n}\sum_{i=1}^{q_r(y)}(1 - d_r(y_i, \overline{y}))^\alpha \qquad \text{where} \qquad d_r(y_i, \overline{y}) = \frac{y_i}{z_r(\overline{y})}, \tag{2}$$

where the only difference with A_α is the definition of the poverty threshold.

Third, the *Atkinson-Bourguignon* overall poverty measures O_α are defined as (Atkinson and Bourguignon, 2001):[4]

$$O_\alpha(y) \coloneqq \frac{1}{n}\sum_{i=1}^{q(y)}(1 - d_{ar}(y_i, \overline{y}))^\alpha \qquad \text{where} \qquad d_{ar}(y_i, \overline{y}) = \frac{y_i}{\max\{z_a, z_r(\overline{y})\}}, \tag{3}$$

[4] Atkinson and Bourguignon (2001) propose a more general family of overall measures. Equation (3) corresponds to the subfamily that they consider in the empirical application. Their alternative measures also violate our normative assumption.

where $\alpha \geq 0$ is the poverty aversion parameter. Measures O_α identify the poor using a poverty line that is the upper-contour of the two poverty lines. This poverty line is absolute in low-income countries and relative in high-income countries. As we show below, measures O_α violate our normative assumption.

Fourth, the *hierarchical* overall poverty measures P^λ are defined as (Decerf, 2017):

$$P^\lambda(y) := \frac{1}{n}\sum_{i=1}^{q(y)}\big(1 - d^\lambda(y_i, \overline{y})\big), \tag{4}$$

where individual i's poverty contribution is $1 - d^\lambda(y_i, \overline{y})$ and

$$d^\lambda(y_i, \overline{y}) := \begin{cases} \lambda \frac{y_i}{z_a} & \text{if } \; y_i < z_a, \\ \\ \lambda + (1-\lambda)\frac{y_i - z_a}{z_r(\overline{y}) - z_a} & \text{if } \; z_a \leq y_i < z_r(\overline{y}), \end{cases} \tag{5}$$

and where parameter $\lambda \in [0, 1]$ tunes the priority given to an individual who is absolutely poor over an individual who is only relatively poor (see below for the interpretation of this key parameter).[5] As we graphically illustrate below, all measures P^λ satisfy our normative assumption.

2.3 Disagreement between absolute and relative measures

In this section, we discuss the implications of our normative assumption for overall poverty comparisons of two distributions for which absolute measures disagree with relative measures. We say that there is a *disagreement* between absolute and relative measures on two distributions when these measures draw opposite evaluations of the distributions, *i.e.* either $A_\alpha(x) > A_\alpha(y)$ and $R_\alpha(x) < R_\alpha(y)$, or $A_\alpha(x) < A_\alpha(y)$ and $R_\alpha(x) > R_\alpha(y)$.

We present our analysis using a stylized example for which the absolute threshold is set at \$1.9 a day (*i.e.* the extreme poverty threshold of the World Bank) and the relative threshold is set at half mean income.[6] Consider distributions x and y shown in Table 1. Both distributions feature three individuals. Individual 1 is absolutely poor, individual 2 is only relatively poor and individual 3 is non-poor. Distribution y is obtained from x by a particular form of "unequal growth". The income of each individual i is larger in y than in x, which yields a mean income in y (\$10) twice as large as the mean income in x (\$5). Yet, the income growth from x to y is not equi-proportional. The income of the non-poor individual 3 is more than doubled while the incomes of the poor individuals 1 and 2 grow at a slower pace.

[5]This family implicitly assumes a poverty aversion $\alpha = 1$. When $\alpha = 1$, the condition under which overall poverty comparisons are independent of the value chosen for λ is simple (see Proposition 2). We discuss the impact of using $\alpha \neq 1$ at the end of Section 2.4.

[6]Our example assumes a strongly relative line but it is straightforward to adapt our reasoning to the case of a weakly relative line.

Table 1: Disagreement over the comparison of x and y

	$i=1$	$i=2$	$i=3$	z_a	z_r
Distribution x	1.6	2	11.4	1.9	2.5
Distribution y	1.8	3	25.2	1.9	5

Note: We set $z_a = 1.9$ and $z_r(\overline{y}) = 0.5\overline{y}$ where $\overline{y}$ is mean income.

When considering gap-sensitive poverty measures ($\alpha > 0$), there is a disagreement between absolute and relative measures over these two distributions that have different income standards: $A_\alpha(x) > A_\alpha(y)$ and $R_\alpha(x) < R_\alpha(y)$.[7] They disagree because they provide different comparisons of individual situations across distributions having different income standards. In our framework, the situation of any individual i is defined by her *bundle* $(y_i, \overline{y})$. Each additive poverty measure implicitly defines a complete ranking of individual bundles, summarized by its *iso-poverty map* (IPM). An iso-poverty map is a collection of *iso-poverty curves*, which are defined as the set of all individual bundles associated to a given value of poverty contribution.

Any iso-poverty curve of A_α and R_α is the set of bundles associated to a given value of normalized income. For A_α, the normalized income only depends on own income and all iso-poverty curves are flat lines, as illustrated by its IPM represented in Figure 1.a for the case $\alpha > 0$. For R_α, the normalized income is own income divided by the relative poverty threshold, which increases with the income standard. As a result, all iso-poverty curves are homothetic to the relative line, as illustrated by its IPM represented in Figure 1.b for the case $\alpha > 0$.[8]

There is more absolute poverty in x than in y because for the absolutely poor individual 1 we have that $x_1 < y_1$. Hence, individual 1's normalized income (with respect to the *absolute* threshold) is smaller in x than in y. Therefore, individual 1 is on a lower iso-poverty curve of A_α in x than in y. In contrast, there is less relative poverty in x than in y because the incomes of individuals 1 and 2 do not grow as fast as the income standard. This implies that their normalized incomes (with respect to the *relative* threshold) are larger in x than in y. Therefore, individuals 1 and 2 are on a higher iso-poverty curve of R_α in x than in y.

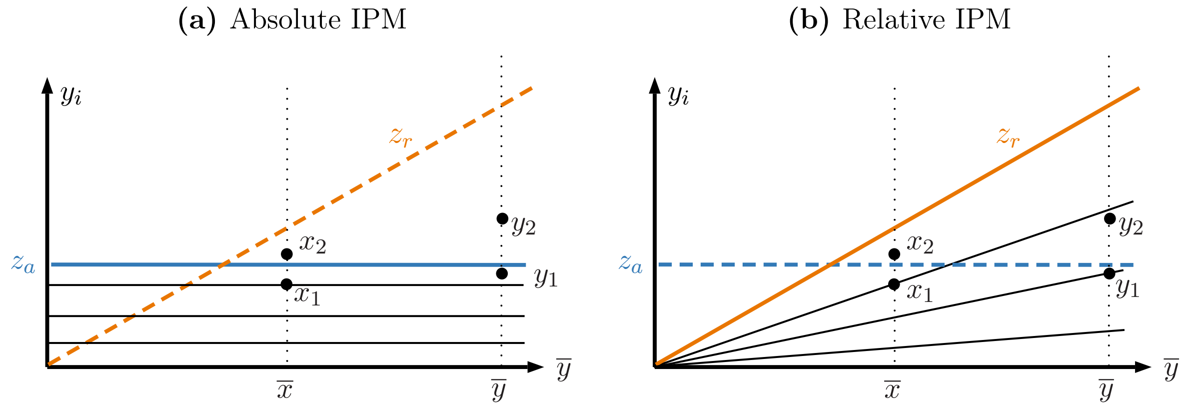
We turn now to overall poverty measures. There is less overall poverty as measured by O_α in x than in y when $\alpha > 0$. The reason is easily understood by looking at the IPM of O_α, which is illustrated in Figure 2.a for the case $\alpha > 0$.[9] This IPM is defined by

[7] In Appendix A.3 we provide another example based on a disagreement between head-count ratios A_0 and R_0.

[8] The IPMs of A_0 and R_0 are slightly different than those illustrated in Figure 1 because all iso-poverty curves below the poverty threshold form a "thick" iso-poverty curve when $\alpha = 0$.

[9] The IPM of O_α is the same for all $\alpha > 0$, but the IPM of O_0 is different because all its iso-poverty curves form a "thick" iso-poverty curve. Different values α define different numerical representations of this IPM.

Figure 1: Distribution x has higher absolute poverty but lower relative poverty than y.



Note: The black lines are iso-poverty curves. These lines reveal how different bundles $(y_i, \overline{y})$ are implicitly compared across distributions with different income standards.

function d_{ar}, which computes the normalized income with respect to the largest poverty threshold. As a result, this IPM corresponds to the IPM of absolute measures in very low-income countries ($z_a > z_r$) and corresponds to the IPM of relative measures in higher income countries ($z_a < z_r$). When comparing distributions x and y, measure O_α agrees with R_α because both distributions have a mean income large enough for the relative threshold to be larger than the absolute threshold.

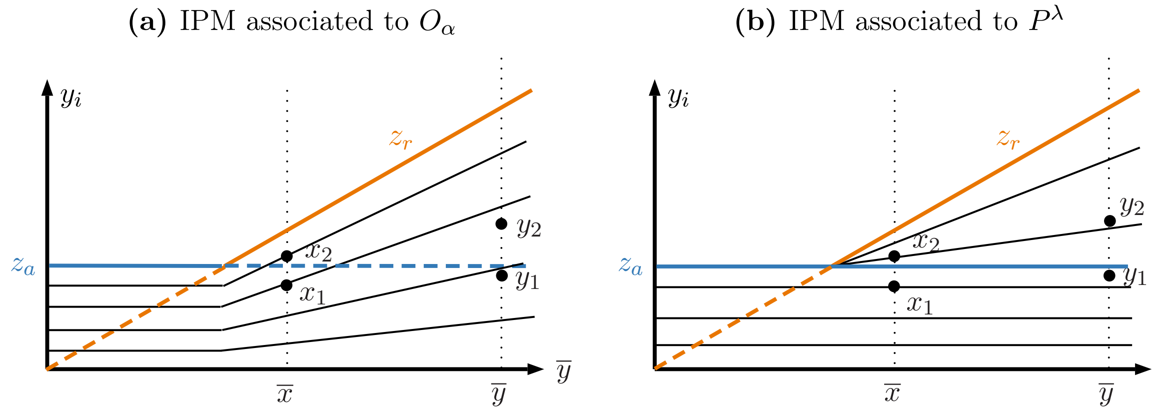
Importantly, the IPM of O_α reveals that this overall measure violates our normative assumption. As illustrated using Figure 2.a, its iso-poverty curves cross the absolute threshold. When $\alpha > 0$, the contribution to O_α of individual 1 in x is smaller than the contribution to O_α of individual 2 in y, even if the former is absolutely poor and the latter is not.[10]

In contrast, there is more overall poverty as measured by P^λ in x than in y, for all values of λ. The reason is easily understood by looking at the inter-personal comparisons inherent to P^λ. All measures P^λ are associated to the same IPM, which is illustrated in Figure 2.b.[11] First, as for measure A_α, all the iso-poverty curves below the absolute threshold are flat. The reason is that the poverty contribution of an absolutely poor individual only depends on her individual income. Thus, the contribution to P^λ of individual 1 is larger in x than in y. Importantly, this also implies that no iso-poverty curve "crosses" the absolute threshold. That is, no iso-poverty curve has some of its bundles below the absolute threshold and some of its bundles above the absolute threshold. Hence, an absolutely poor individual always contributes more to P^λ than an individual who is

[10]When $\alpha = 0$, the contribution of individual 1 in x is the same as the contribution to O_α of individual 2 in y.

[11]Different values of the parameter λ define different numerical representations of this IPM. Strictly speaking, the IPMs for P^0 and P^1 are slightly different because each of these measures has a "thick" iso-poverty curve. In the case of P^0 all the iso-poverty curves below z_a form a "thick" iso-poverty curve. In the case of P^1 all the iso-poverty curves above z_a form a "thick" iso-poverty curve.

Figure 2: Under our normative assumption (P^λ), distribution x has a larger overall poverty than y.



Notes: The black lines are iso-poverty curves. These lines reveal how different bundles $(y_i, \overline{y})$ are implicitly compared across distributions with different income standards.

only relatively poor.[12] This shows that measure P^λ satisfies our normative assumption. Second, the IPM reveals that the contribution to P^λ of individual 2 is also larger in x than in y.[13] At any bundle above the absolute threshold, the slope of the iso-poverty curve associated to P^λ is less steep than the slope of the iso-poverty curve associated to R_α. Iso-poverty curves associated to P^λ make a trade-off between the absolute and relative aspects of income, while iso-poverty curves associated to R_α only capture the relative aspect.

This stylized example shows that overall poverty comparisons depend on the inter-personal comparisons made across societies with different income standards. Our normative assumption constrains these inter-personal comparisons by giving some precedence to absolutely poor individuals. Unlike O_α, overall measure P^λ satisfies this assumption.

2.4 Unambiguous overall comparisons with P^λ

In this section, we characterize the conditions under which we can draw overall poverty comparisons with P^λ that are independent of the priority parameter λ. As we show below, it is possible to draw such unambiguous comparisons with P^λ even for some pairs of distributions for which A_α and R_α disagree.

Parameter λ has a key normative interpretation. It tunes the priority given to an individual who is absolutely poor over an individual who is only relatively poor. Mathe-

[12] This can also be verified mathematically by looking at the contribution function d^λ.

[13] Observe that measure P^λ can be given a welfarist interpretation in which the underlying utility function is expressed in Equation (5). According to the utility function $d^\lambda(y_i, \overline{y})$, concerns about relative deprivation emerge only when the income standard is above some critical level and when own income is above the absolute threshold. Under this interpretation, individuals prefer to have the possibility of minimally satisfying their basic needs, even if having this possibility increases the cost of social participation. Ravallion and Lokshin (2010) provide some empirical evidence that absolute consumption needs dominate welfare at very low levels of consumption.

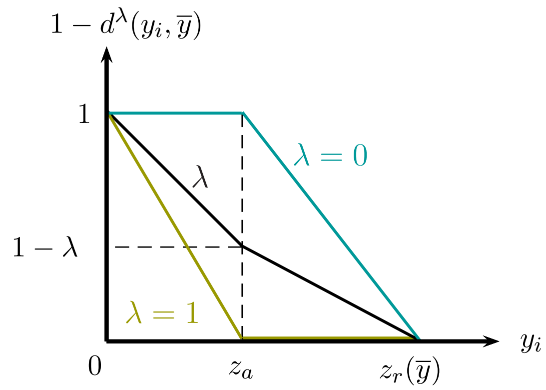
matically, this parameter tunes the marginal poverty contributions of these two types of poor individuals. Letting i be absolutely poor and j only relatively poor, we get from Equation (5) that

$$\frac{\partial d^\lambda(y_i,\overline{y})}{\partial y_i} = \frac{\lambda}{z_a} \qquad \text{and} \qquad \frac{\partial d^\lambda(y_j,\overline{y})}{\partial y_j} = \frac{1-\lambda}{z_r(\overline{y})-z_a}.$$

Thus, when i earns an additional ϵ of income, her contribution to poverty decreases by $\epsilon\frac{\lambda}{z_a}$, regardless of her exact income.[14] The larger λ, the larger is the decrease in her contribution. In contrast, when j earns an additional ϵ of income, her contribution decreases by $\epsilon\frac{1-\lambda}{z_r(\overline{y})-z_a}$, regardless of her exact income. The larger λ, the smaller is the decrease in her contribution. When λ is large (close to 1), giving an additional ϵ to an absolutely poor individual reduces P^λ much more than giving it instead to an only relatively poor individual. The support of parameter λ contains all possible views on the respective priority that could be given to absolutely poor individuals. For the extreme case $\lambda = 1$, absolutely poor individuals have infinite priority because the additional ϵ is infinitely more poverty reducing when given to an absolutely poor individual. For the other extreme case $\lambda = 0$, individuals who are only relatively poor have infinite priority over absolutely poor individuals.

Figure 3 graphically illustrates the impact of parameter λ on the shape of the contribution function at a fixed level of income standard. As the graph for $\lambda = 1$ reveals, P^1 gives infinite priority to the absolutely poor because the contribution of the only relatively poor is constant in own income. Then, the graph for $\lambda = 0$ shows that P^0 gives infinite priority to the only relatively poor because the contribution of the absolutely poor is constant in own income.

Figure 3: Contribution as a function of income y_i, at a fixed income standard $\overline{y}$.



In general, when there is a disagreement between absolute and relative measures, the

[14] Recall that, when $z_r > z_a$, the contribution function d^λ is continuous in y_i at z_a. This implies that the reduction of the contribution d^λ that results from raising the income of the absolutely poor i to z_a becomes arbitrarily small when $y_i \to z_a$.

overall poverty comparison with P^λ depends on the value taken by parameter λ. This is for instance illustrated in Appendix A.2. However, there are pairs of distributions for which A_α and R_α disagree but the overall poverty comparison with P^λ is *unambiguous*. This is illustrated with the stylized example given in Table 1. In that example, the unambiguous comparison follows from the fact that all poor individuals are on a higher iso-poverty curve of P^λ in y than in x. In this particular sense, distribution y first-order stochastically dominates distribution x (Atkinson, 1987). Thus, regardless of the value given to parameter λ, there is more overall poverty (as measured by P^λ) in x than in y. Proposition 1 formalizes the possibility to make unambiguous overall comparisons with P^λ even when absolute and relative measures disagree.

Proposition 1. *There exist distributions $x, y \in Y$ for which $A_\alpha(x) > A_\alpha(y)$ and $R_\alpha(x) < R_\alpha(y)$ for all $\alpha \geq 0$ and for which $P^\lambda(x) > P^\lambda(y)$ for all $\lambda \in [0,1]$.*

Proof. The proof is in Appendix A.3. ■

Note that it is not necessary that all bundles move onto higher iso-poverty curves in order to have an overall poverty comparison that does not depend on the priority parameter.[15]

The necessary and sufficient condition under which an overall poverty comparison does not depend on the value chosen for λ follows from Proposition 2.

Proposition 2. *For any two distributions $x, y \in Y$, either we have $\frac{P^0(x)}{P^0(y)} \leq \frac{P^\lambda(x)}{P^\lambda(y)} \leq \frac{P^1(x)}{P^1(y)}$ for all $\lambda \in [0,1]$ or we have $\frac{P^0(x)}{P^0(y)} \geq \frac{P^\lambda(x)}{P^\lambda(y)} \geq \frac{P^1(x)}{P^1(y)}$ for all $\lambda \in [0,1]$.*

Proof. The proof is in Appendix A.4. ■

Proposition 2 directly implies that checking whether an overall poverty comparison is independent of λ only requires computing P^λ for the two extreme values of λ.

Corollary 1. *$P^\lambda(x) \geq P^\lambda(y)$ for all $\lambda \in [0,1]$ if and only if $P^0(x) \geq P^0(y)$ and $P^1(x) \geq P^1(y)$.*

Corollary 2. *$\frac{P^\lambda(y)}{P^\lambda(x)} \leq \frac{1}{2}$ for all $\lambda \in [0,1]$ if and only if $\frac{P^0(y)}{P^0(x)} \leq \frac{1}{2}$ and $\frac{P^1(y)}{P^1(x)} \leq \frac{1}{2}$.*

The easy-to-use conditions obtained in Proposition 2 are the consequence of the linear expression of P^λ. We discuss in Appendix A.1 the implications of considering hierarchical indices based on non-linear expressions.

[15] Poverty contributions to P^λ are linear in own income. Consider two poor individuals 1 and 2 whose incomes are on the same side of the absolute threshold. If we increase the income of individual 1 by an ϵ and decrease the income of individual 2 by less than ϵ while keeping the income standard constant, then P^λ is (weakly) decreased regardless of λ. For instance, distribution $(1, 1, 4, 34)$ has unambiguously less overall income poverty than distribution $(0.8, 1.1, 4, 34.1)$, even if the bundle of individual 2 is on a lower iso-poverty curve under the former distribution.

3 Support for normative assumption

3.1 Survey design

To test whether our normative assumption receives support from the general population, we conducted an online survey in the US, the UK and South Africa. In each country, we collected data from a sample of 385 respondents representative of the national population in terms of region, age and gender, resulting in a total sample of 1155. Survey responses were collected between March 25 and April 17, 2021 using Qualtrics.

Respondents were faced with two hypothetical scenarios, a simple and a more complex one. In each of these scenarios there are two individuals (a and b) living in two different countries (A and B). Person a is absolutely poor while Person b is only relatively poor, and they are respectively described as not having enough income to satisfy her basic needs or to participate in the everyday activities of her country. We asked respondents to rank these two individuals according to their degree of poverty.[16] In the simple scenario, we additionally asked respondents which situation they would prefer for themselves.[17]

The simple and complex scenarios differ in several respects. First, in the simple scenario we explicitly mention whether an individual does not have enough income to either satisfy her basic needs or to participate in the everyday activities of her country. In the complex scenario instead, respondents have to identify the poverty status from individual income levels and poverty thresholds. Second, in the simple scenario Person a is not relatively poor while in the complex scenario she is both absolute and relatively poor.

3.2 Results

Table 2 displays the distribution of responses by their ranking of poverty types for both scenarios. Columns 1 and 2 display the results for the simple and complex scenarios respectively including all responses. We observe that in both scenarios almost 60% of respondents believe that the absolutely poor individual is poorer than the relatively poor individual. In both cases, this share is three times larger than for any other option.

In Column 3 we focus on those respondents who provided consistent answers across the simple and complex questions. We classify a response as consistent if the respondent selects a similar choice in both scenarios: either the same person (a or b) as poorer, both individuals as equally poor, or none as poor. These responses are probably of higher quality than the inconsistent ones. For instance, consistent respondents are likely to have put more effort into selecting their answers or to have higher cognitive skills. Our results

[16] The order of the options within each question was randomized.
[17] The transcript of these survey questions can be found in online Appendix B.

indicate that consistent respondents are slightly more educated than inconsistent ones.[18] When we focus on consistent respondents, we observe that more than 80% of them believe that the absolutely poor individual is poorer than the relatively poor in both scenarios.

In each country surveyed, the majority of respondents gives support to our normative assumption. Specifically, in each country more than 50% (70%) of all (consistent) respondents believe that the absolutely poor individual is poorer, and the share of respondents who choose any other option is at least half of this. The main difference across countries is that there is a slightly higher support for our normative assumption in the US and the UK than in South Africa (see Tables C.1, C.3 and C.5 in the online Appendix.)

Table 2: Opinions about severity of poverty type (%).

Who is poorer	All respondents		Consistent respondents
	Simple scenario (1)	Complex scenario (2)	Both scenarios (3)
Absolutely poor	59.8	59.0	81.5
Relatively poor	11.2	19.8	5.8
Equally	18.5	19.4	11.7
None	10.5	1.8	1.0
Observations	1155	1155	572

Notes: The table shows the proportion of survey respondents who consider that either an absolutely poor individual is poorer than an only relatively poor individual, the contrary, or that both individuals are equally poor. We do not include the remaining share of respondents who choose none of the above alternatives. Column 1 (2) considers all responses to the simple (complex) question, and Column 3 restricts the sample to those respondents who provide consistent answers between the simple and complex scenarios. Source: own survey.

So far, we have analyzed the opinions about the severity of different types of poverty. We turn now to preferences. For the simple scenario, we asked respondents whether they would prefer to be the only absolutely poor person, the only relatively poor person, or whether they would be indifferent. Almost 80% of all respondents state that they would prefer to be only relatively poor over only absolutely poor. This share goes up to 86% among consistent respondents (see Table 3). These results are similar across countries (see Tables C.2, C.4 and C.6 in the online Appendix.) Taking all the results from the survey together, we can conclude that they give substantial support to our normative assumption.

[18] This is particularly the case for the UK: 33% of inconsistent respondents have a Bachelor's degree or higher education, while this share goes up to 48% among consistent respondents.

Table 3: Preferences over poverty type. Simple scenario. (%).

Preference	All respondents (1)	Consistent respondents (2)
Only relatively poor	79.9	86.5
Only absolutely poor	9.2	5.8
Indifferent	10.9	7.7
Observations	1155	572

Notes: The table shows the proportion of survey respondents who state that they would prefer to be either absolutely poor or relatively poor, or that they are indifferent between the two. Column 2 restricts the sample to those respondents who provide consistent answers between the simple and complex scenarios.
Source: own survey.

4 Data and parameters

4.1 Poverty data

Our source of data is PovcalNet,[19] an online tool of the World Bank whose main goal is to replicate the Bank's poverty estimations. PovcalNet offers income or consumption data from more than 160 countries in the world from 1981 to 2015. We use data from 1990 until 2015. We estimate poverty for each reference year defined by the World Bank, these being designed to perform multi-country aggregations since surveys are conducted in different years across countries.[20] We take 1990 as our base year because it was the reference year used for the objective of halving global extreme poverty by 2015 (one of the United Nations' Millennium Development Goals). We include all countries that have information in both 1990 and 2015.[21] The final sample includes 160 countries, among which three have data for rural and urban areas separately. This gives a total of 163 units of analysis.

One of the main advantages of PovcalNet is that it provides poverty estimates that are internationally comparable. In order to allow for cross-country comparisons, the World Bank translates the survey data using the 2011 PPP exchange rates for household consumption from the International Comparison Program.

[19]PovcalNet: the on-line tool for poverty measurement developed by the Development Research Group of the World Bank can be found in: http://iresearch.worldbank.org/PovcalNet/povOnDemand.aspx.

[20]The reference years available between 1990 and 2015 are: 1990, 1993, 1996, 1999, 2002, 2005, 2008, 2010, 2011, 2012, 2013 and 2015. When we analyze a specific country, such as (urban) China, we include all years.

[21]We exclude the following countries with missing information in 1990 and/or 2015: Kosovo, Maldives, Sao Tome and Principe, South Sudan, Timor-Leste and Venezuela. See list of countries included in the sample in Table A.1 in the online Appendix.

4.2 Poverty lines

Estimating poverty with P^λ requires selecting both an absolute line (z_a) and a relative line (z_r). We consider several pairs of poverty lines (see Section 5.2.2), but we mostly focus on our preferred pair of lines. In our main pair of lines, the absolute threshold is set at $1.9 per person per day, in 2011 PPP. This has been the official extreme poverty threshold of the World Bank since 2015 (Ferreira et al., 2016). Our main relative threshold, in turn, is set at half mean income in each country. Selecting a relative line that is mean-sensitive instead of median-sensitive is a conservative assumption. This choice magnifies the relative component of our overall poverty measures because mean income is significantly larger than median income in most countries. Also, many countries saw their mean income increase faster than their median income over 1990-2015. Therefore, if the reduction in absolute poverty more than compensates the increase in relative poverty under a mean-sensitive line, it is very likely also to hold when changing the income standard to median income. Finally, a slope equal to 0.5 is standard for mean-sensitive relative lines.[22]

5 Empirical results

We present three sets of results. First, we show that global overall poverty measured by P^λ has been halved over the period 1990-2015, independently of the value chosen for the priority parameter. Second, we show that this result is robust to using alternative population weights and alternative poverty lines. Finally, we compare our results to those obtained by alternative standard measures in terms of the magnitude of poverty change.

5.1 Evolution of overall poverty by P^λ

We start by analyzing the change in poverty between 1990 and 2015 in a small set of countries (see Table 4).[23] These countries were selected for illustrative purposes. Except for Pakistan, they have all experienced a decrease in absolute poverty (both by A_0 and A_1) and an increase in relative poverty (both by R_0 and R_1). Altogether, these countries cover more than 46% of the sample population size over every year from 1990-2015. In particular, China, India, Indonesia and Pakistan are the top four most populous countries in the developing world.

[22] In Section 5.2.2, we use an alternative (higher) mean-sensitive relative line (*i.e* $z_r = 0.4 + 0.5\overline{y}$), which obviously yields higher levels of poverty when combined with the same absolute line. However, given that this alternative relative line increases at a smaller rate when mean income increases, its poverty reduction estimates are *a priori* not necessarily more conservative than those of our main specification.

[23] As our data source provides separate consumption distributions for rural and urban areas for China, India and Indonesia, we analyze them separately. The relative threshold in rural (resp. urban) areas are computed using the income standard in rural (resp. urban) areas.

Table 4 displays the values of mean income per capita, inequality and poverty as measured by various indices in 2015 relative to 1990 for each country. Consider for instance the row corresponding to urban China. We observe that urban China has experienced a sharp increase both in mean income per capita and inequality as measured by the Gini index over this period (see Columns 1 and 2). The former led to a sharp decrease in absolute poverty as measured by A_0 and A_1 (see Columns 4 and 5). In turn, the increase in inequality led to an increase in relative poverty as measured by R_0 and R_1 (see Columns 6 and 7). This shows that, when taking $\alpha = 0$ or $\alpha = 1$, the absolute measures disagree with the relative measures on the evolution of poverty in urban China (as indicated in Column 8). In turn, the overall poverty measures P^1 (equivalent to A_1) and P^0 have also declined over 1990-2015 (see Columns 3 and 4). As both P^1 and P^0 have decreased, from Corollary 1 we can conclude that overall poverty measured by P^λ has been reduced in urban China, independently of the value chosen for the priority parameter (as indicated in the last Column). In this sense, the decrease in absolute poverty more than compensates the increase in relative poverty. Moreover, as both P^1 and P^0 have been at least halved over the period, we can conclude from Corollary 2 that P^λ has been (at least) halved in urban China, independently of the value chosen for the priority parameter.

Table 4: Statistics and poverty for selected countries. Values of 2015 relative to 1990.

	Mean inc. (1)	Gini (2)	P^0 (3)	$P^1 = A_1$ (4)	A_0 (5)	R_0 (6)	R_1 (7)	Dis. (8)	Unam. (9)
Bangladesh	1.51	1.24	0.40	0.29	0.39	1.37	1.53	Yes	Yes
China									
Rural	4.68	1.09	0.11	0.01	0.02	1.39	1.79	Yes	Yes
Urban	5.25	1.41	0.28	0.01	0.01	2.41	3.60	Yes	Yes
India									
Rural	1.64	1.04	0.31	0.20	0.31	1.12	1.11	Yes	Yes
Urban	1.71	1.17	0.52	0.21	0.30	1.24	1.44	Yes	Yes
Indonesia									
Rural	2.64	1.26	0.18	0.05	0.09	1.56	2.45	Yes	Yes
Urban	2.37	1.23	0.51	0.09	0.15	1.50	2.15	Yes	Yes
Jamaica	1.51	1.11	1.03	0.49	0.40	1.18	1.32	Yes	No
Pakistan	2.19	N/A	0.17	0.04	0.09	0.83	0.62	No	Yes
World	1.43	N/A	0.41	0.24	0.28	1.07	1.01	Yes	Yes

Notes: Mean income per capita is expressed in PPP$ per month. A_0 (A_1) and R_0(R_1) are defined as in Equations (1) and (2) with $\alpha = 0$ ($\alpha = 1$). P^0 (P^1) is defined as in Equation (4) with $\lambda = 0$ ($\lambda = 1$). The column labeled "Dis." indicates whether there is a disagreement between A_0 and R_0 on the poverty change between 2015 and 1990. For these set of countries A_1 (R_1) evolves in the same direction as A_0 (R_0). The last column labeled "Unam." identifies whether the poverty change according to P^λ is independent of the value of λ. For some countries, the Gini is not available for 1990 and/or 2015. We impute the Gini when there is survey data available in a window of 10 years around each reference year. The imputation concerns the following countries and reference years in the table (we indicate the survey year used to input the Gini between brackets): Bangladesh in 1990 (1981), 2015 (2010), India in 1990 (1983).

The evolution of poverty in urban China is not an exception as many countries, especially in the developing world, experienced both a strong growth and an increase in within-country inequality over the period (Bourguignon, 2015; Milanovic, 2016; Anand and Segal, 2008; Ravallion, 2014). Several other cases presented in Table 4, namely Bangladesh, rural China, rural and urban India and Indonesia, experience a similar evolution: the absolute measures disagree with the relative measures but overall poverty measured by P^λ is unambiguously reduced. In four out of these six cases, we can conclude that P^λ has been unambiguously halved. In the remaining cases, *i.e.* urban India and urban Indonesia, whether P^λ has been halved or not depends on the priority parameter (P^0 has not been halved over the period) but the reduction is at least 48% for any λ. The last two countries in the table provide examples of alternative trends in poverty. In Pakistan, there was no increase in relative poverty but the strong decrease in absolute poverty has led overall poverty measured by P^λ to be divided by a factor at least larger than five. In Jamaica, in turn, the decrease in absolute poverty was not large enough to offset the increase in relative poverty, leading to a slight increase in P^λ when the priority given to absolutely poor individuals is sufficiently low (as revealed by P^0).

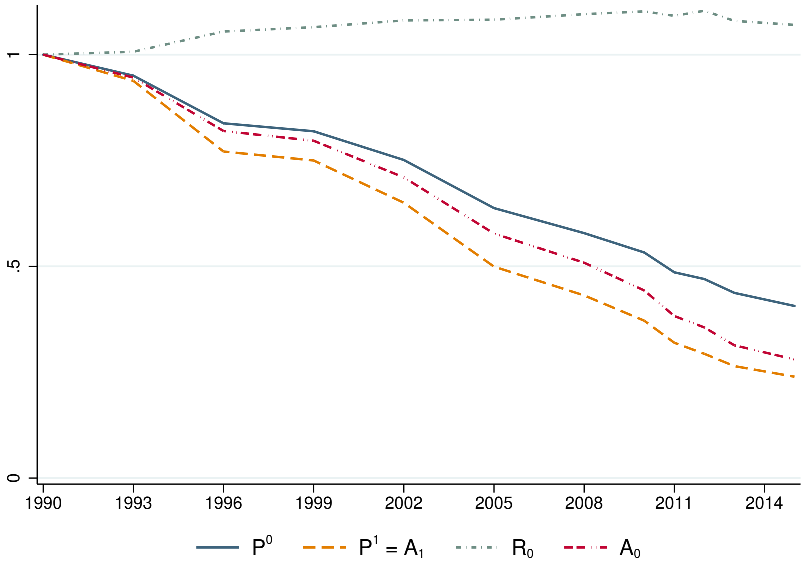
We turn now to the evolution of global poverty. To keep the exposition simple, for the absolute and relative measures we focus on the head-count ratios A_0 and R_0 and, whenever relevant, we mention whether the results also hold (qualitatively) for $\alpha = 1$. Figure 4 shows the evolution of global poverty (relative to 1990) by P^λ, A_0 and R_0 (see also the last row of Table 4). The absolute measure A_0 disagrees with the relative measure R_0: A_0 has declined by 72% while R_0 has increased by 7%.[24] The overall poverty measure P^1, which gives infinite priority to absolutely poor individuals, has declined by 76%. Finally, the overall poverty measure P^0, which gives infinite priority to relatively poor individuals, has declined by 59%. Thus, there is an unambiguous reduction in global poverty measured by P^λ. Moreover, P^0 provides the lower bound for this overall poverty reduction, which is larger than 50%. By Corollary 2, we can conclude that P^λ has been halved over the period, independently of the priority assigned to the absolutely poor.

The trends in global poverty are similar to those observed in the developing world, which concentrates most of the reduction in absolute poverty over this period (see Figure C.1 in the online Appendix). If we look at the evolution of poverty by regions[25], we observe that the decline in P^λ is mostly driven by (populous) regions with large initial poverty. These are mainly East Asia and Pacific, South Asia and Sub-Saharan Africa, which respectively explain 53, 23, and 19% of global P^1 in 1990 and 47, 26, and 14% of global P^0 in 1990. Figures A.2a to A.2c in the Appendix show the evolution of poverty in

[24]The results are qualitatively similar if we focus on poverty-gaps instead: A_1 has declined by 76% while R_1 has increased by 1%.

[25]The World Bank divides the world into seven regions: (1) East Asia and Pacific, (2) Europe and Central Asia, (3) Latin America and the Caribbean, (4) Middle East and North Africa, (5) North America, (6) South Asia, and (7) Sub-Saharan Africa.

Figure 4: Evolution of global poverty relative to 1990.



Notes: The graph plots the evolution of global poverty as measured by different indices for all reference years until 2015 relative to 1990.

these three regions (we present the remaining regions in Figures C.2a to C.2d in the online Appendix). All regions except from North America have experienced an unambiguous decline in P^λ over 1990-2015. Moreover, P^λ has been unambiguously halved in East Asia and Pacific, and South Asia.

5.2 Robustness

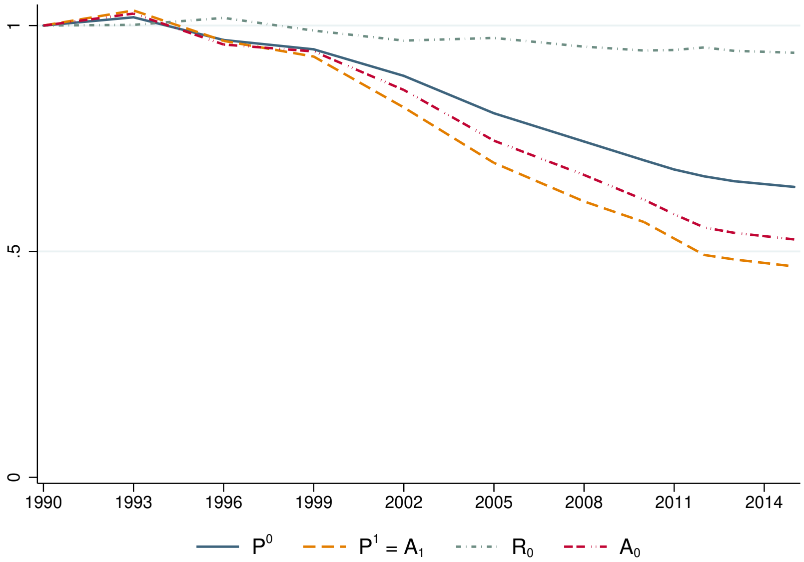
In this section, we study the robustness of our results in two different ways. First, we study robustness to population weights and verify that the results are not fully driven by a few major countries. Second, we study whether our results are robust to alternative definitions of the poverty lines.

5.2.1 Robustness to population weights

One potential concern about our analysis is whether the reduction in P^λ is completely driven by the evolution of poverty in one or two large countries. In order to assess this, we perform two robustness checks. First, we exclude China and India from the sample. Second, we fully ignore population weights and compute the number of countries for which we can conclude that P^λ has decreased (resp. has been halved) regardless of the priority parameter.

China and India represent together almost 40% of our sample population size. Also, they have both experienced a strong reduction in P^λ for all λ. We first analyze whether the global decline in P^λ also holds when we exclude these two countries. Figure 5 shows that even when these large economies are removed, both P^0 and P^1 have significantly decreased. When removing China and India, absolute poverty measured by A_0 decreases by 46% (instead of 72%) and overall poverty measured by P^λ decreases by at least 36% (instead of 59%) (see Table A.4 in the Appendix).[26] Hence, these two countries alone do not completely drive our result.

Figure 5: Evolution of global poverty excluding China and India relative to 1990.



Notes: The graph plots the evolution of global poverty excluding China and India as measured by different indices for all reference years until 2015 relative to 1990.

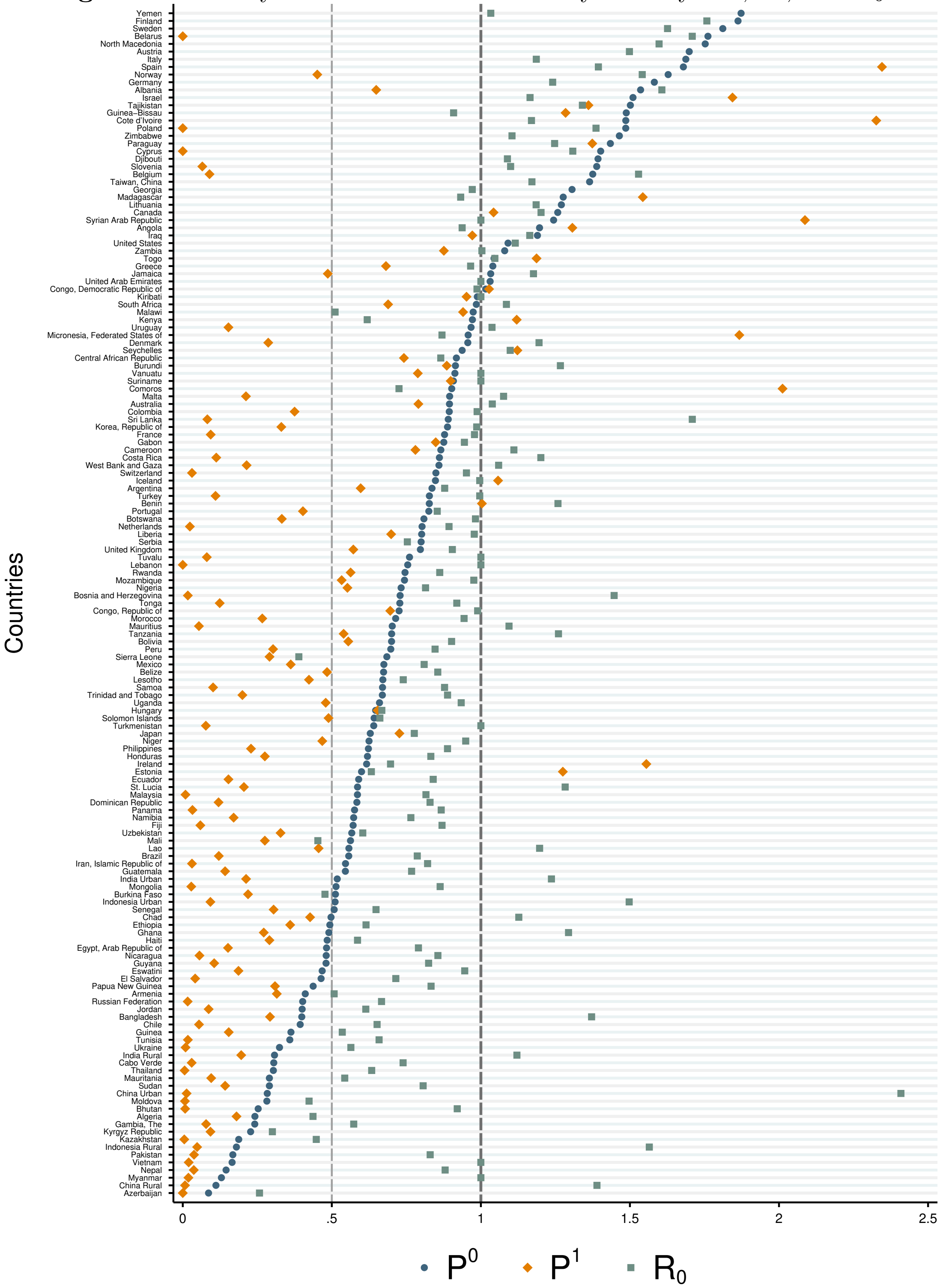
Second, we study the robustness of our results to ignoring population weights. Figure 6 displays the ratio of P^0, P^1 and R_0 in 2015 relative to 1990 for each country in our sample.[27] Countries are ordered in descending order of the value of P^0 in 2015 relative to 1990. We can easily observe that except for a few countries at the top of the graph, most countries experience a decrease in P^0. Moreover, most of them also experience a decrease in P^1, the other extreme member of our family. These two observations together imply that P^λ is reduced in many countries, independently of the priority parameter.

More precisely, we can compute the fraction of countries for which P^λ has been reduced and the fraction for which it has been halved, independently of the priority parameter. To do so, we perform all within-country pairwise poverty comparisons between 1990 and 2015. For each pairwise comparison, we first identify whether there is a disagreement between A_0 and R_0. We observe that A_0 and R_0 have evolved in opposite directions in almost 30% of the cases. Results are similar when we compare A_1 and R_1 (35% of disagreements) (see Table A.5 in the Appendix). Moreover, we observe that P^λ has unambiguously declined in 67% of countries and has been unambiguously halved in 23% of countries (see Table A.6 in the Appendix).

[26] Table A.4 in the Appendix further shows that P^λ has been unambiguously reduced when excluding only China or India. Figures A.1a and A.1b in the Appendix display the evolution of poverty in East and South Asia (China and India's respective regions) excluding them.

[27] The graph looks similar if we use R_1 instead. See Figure C.3 in the online Appendix.

Figure 6: Poverty in 2015 relative to 1990 by country. P^0, P^1, and R_0.



Notes: The graph plots the values of P^0, P^1, and R_0 in 2015 relative to 1990 for each country in the sample. For visualization purposes, values larger than 2.5 are not displayed. We exclude countries with at least two values larger than 2.5. This concerns: Bulgaria, Croatia, Czech Republic, Latvia, Luxembourg, Montenegro, Romania & Slovak Republic. The following countries are displayed but have one value larger than 2.5: Austria, Djibouti, Finland, Georgia, Germany, Italy, Lithuania, North Macedonia, Serbia, Sweden, Taiwan, China, United Arab Emirates, United States, Yemen & Zimbabwe. Finally, note that there are several countries in the graph for which R_0 in 1990 is the same as in 2015, and hence the ratio 2015/1990 equals 1. For these countries, PovcalNet has survey data for only one year over the whole period. Thus, to extrapolate the distribution across years they assume equi-proportionate growth. This implies that when R_0 is defined using a strongly relative line, it does not change over time. This affects the following countries: Kiribati, Lebanon, Suriname, Syrian Arab Republic, Turkmenistan, Tuvalu, United Arab Emirates & Vanuatu.

5.2.2 Robustness to poverty lines

We show here that our results still hold for alternative pairs of poverty lines. Table 5 displays the specific combinations of absolute and relative lines that we use. The first five pairs of lines (pairs 1 to 5 in Table 5) all use different relative lines but the same absolute line. The first alternative relative line is similar to our main relative line but is based on median income instead of mean income. The second alternative relative line is also based on median income and has the same gradient as the previous one but in addition it has an intercept of $1. This line, called the Societal poverty line, has been estimated by Jolliffe and Prydz (2017) from regressions of 699 national poverty thresholds against median income. A recent report from the World Bank estimates societal poverty, which corresponds to the head-count ratio below the Societal poverty line (World Bank, 2018). The third alternative relative line has an intercept of 0.4 and a relative gradient of 50% of the mean national income. This line has been estimated from regressions of national poverty thresholds by Ravallion and Chen (2017) (see their Figure 5 panel b). As some authors consider relative lines with a smaller slope parameter (see for instance Atkinson and Bourguignon, 2001), our pair 5 has a slope of 0.33 and an intercept of $1. Finally, our sixth combination of lines sets the absolute line at 3.2 PPP$ a day and uses the relative line of our main specification (pair 1). The absolute threshold of $3.2 a day corresponds to the lower-middle-income international poverty line suggested by the World Bank (see Jolliffe and Prydz, 2016).

Table 5: Global poverty in 2015 relative to 1990 for different pairs of lines and poverty measures.

Pair #	z_a	z_r	Income standard $\bar{y}$	P^0 (1)	$P^1 = A_1$ (2)	O_0 (3)	O_1 (4)	A_0 (5)	R_0 (6)	R_1 (7)
1	1.9	$0.5\bar{y}$	Mean	0.41	0.24	0.56	0.48	0.28	1.07	1.01
2	1.9	$0.5\bar{y}$	Median	0.34	0.24	0.43	0.36	0.28	1.00	0.95
3	1.9	$1 + 0.5\bar{y}$	Median	0.44	0.24	0.61	0.51	0.28	0.62	0.54
4	1.9	$0.4 + 0.5\bar{y}$	Mean	0.45	0.24	0.64	0.55	0.28	0.85	0.81
5	1.9	$1 + 0.33\bar{y}$	Mean	0.39	0.24	0.53	0.44	0.28	0.59	0.53
6	3.2	$0.5\bar{y}$	Mean	0.53	0.35	0.60	0.43	0.48	1.07	1.01

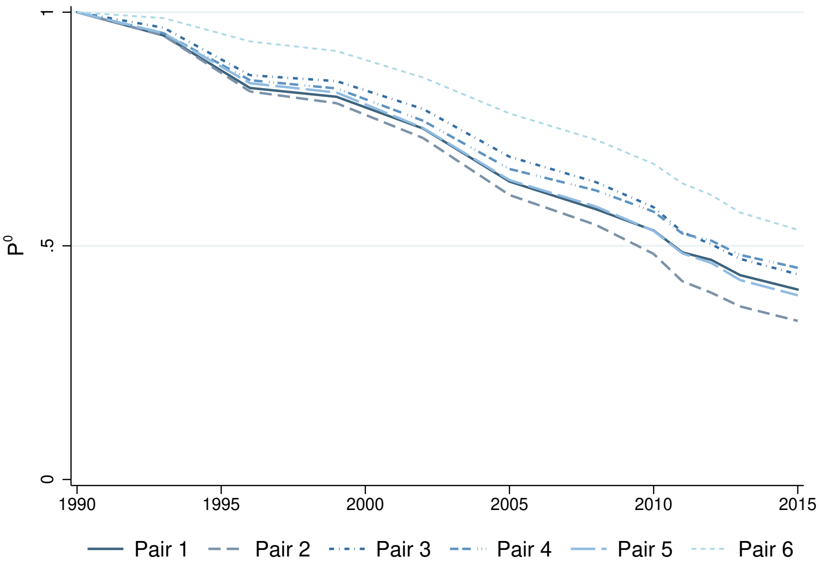
Notes: The table shows the values of different poverty indices in 2015 relative to 1990 for the six pairs of lines considered. We include eight poverty indices setting α or λ equal to 0 and 1 for each of the four families of poverty measures considered.

For all pairs of lines, we observe a continuous decrease in overall measure P^0 over 1990-2015 (see Figure 7). Considering all pairs, the decline in P^0 between 1990 and 2015 ranges from 47% to 66% (see Column 1 of Table 5). Even considering the most conservative pair of lines (pair 6), P^0 decreases by almost 50% between 1990 and 2015.[28]

[28] Columns 2 and 5 of Table 5 respectively display absolute measures A_0 and A_1. We observe that if we raise the absolute threshold from $1.9 to $3.2, A_0 has decreased by 52% (compared to 72% under $1.9 and the same relative line). The differences are smaller if we instead compare A_1 across pairs (A_1

This shows that our main result is robust to using alternative relative lines and almost robust to all pair of lines.

Figure 7: Evolution of P^0 relative to 1990 by line.



Notes: The graph plots the evolution of P^0 relative to 1990 for all pairs of poverty lines as defined in Table 5.

Table C.7 in the online Appendix replicates Table 4 for the same selection of countries using the most conservative pair of lines. All selected countries except from Jamaica (as for the first pair of lines) have experienced a substantial decrease in overall poverty either measured by P^0 or P^1 with this alternative pair. Moreover, for all of the selected countries the decrease in P^λ is independent of the priority parameter.

Finally, considering all countries in our sample individually for our most conservative pair of lines, we observe that P^λ has unambiguously declined in 72% of them. Moreover, P^λ has been unambiguously halved in 20% of them (see Table C.8 in the online Appendix). Again, this shows that even for alternative pairs our results are not fully driven by a small number of large countries.

5.3 Comparison of overall poverty measures

5.3.1 Impact of index versus impact of poverty line

The literature on global poverty measurement has paid more attention to the definition of the poverty lines than to the selection of the poverty index. We can compare the reduction of poverty between 1990 and 2015 across overall poverty measures and all the pairs of poverty lines considered. To quantify this comparison, we compute two mean variances. First, we compute the variance in poverty reduction between the main overall poverty measures (P^0, O_0 and O_1) for each pair of lines and we take the average across

decreases by 65% for pair 6 against 76% for pair 1).

pairs (*"between index variance"*). Second, we compute the variance in poverty reduction between pairs of lines for each index and we take the average across indices (*"within index variance"*).[29] We conservatively ignore P^1 first, even if including it would only increase the variance between indices, because it is formally equivalent to A_1, a purely absolute index. Table 6 shows that the between and within index standard deviations are quite similar: 0.063 and 0.070 respectively.[30] This comparison suggests that in this case the selection of poverty indices affects the magnitude of overall poverty reduction as much as the selection of poverty lines.

Table 6: Impact of indices vs. poverty lines on variance in poverty reduction.

	Between index standard deviation (1)	Within index standard deviation (2)
Excluding P^1	0.063	0.070
Including P^1	0.115	0.064

Notes: The first column of the table displays the standard deviation between indices for each pair of lines averaged across pairs. The second column displays the standard deviation between pairs of lines for each index averaged across indices. The first row considers P^0, O_0 and O_1, while the second row also includes P^1.

5.3.2 Comparison of P^λ with the *Atkinson-Bourguignon* measures

In this section, we compare the results on overall poverty change obtained by P^λ with the alternative approach most commonly used in the literature. Despite the normative appeal of our approach, its empirical relevance largely depends on the extent to which poverty change estimates differ from those obtained using standard measures. The dominant practice in evaluating global overall poverty is to estimate the evolution of the *Atkinson-Bourguignon* overall poverty measure O_0, *i.e.* the head-count ratio below the upper-contour of the absolute and relative lines. This is for instance the approach followed by Chen and Ravallion (2013), Jolliffe and Prydz (2017) and Ravallion and Chen (2017). Thus, we focus our comparative analysis on O_0, and we briefly comment on the comparison with O_1.

If we look again at Table 5, we can compare our estimation of overall poverty reduction (P^λ) with that estimated by O_0. The main takeaway is that, for all pair of lines, the poverty reduction estimated by O_0 does not lie inside our two bounds (P^1 and P^0). Even our most conservative estimation (associated with P^0) finds more poverty reduction than

[29] Precisely, to compute the *"between index variance"* we first compute the variance between the values of P^0, O_0 and O_1 in 2015 relative to 1990 for each pair of lines (*i.e.* variance between Columns (1), (3) and (4) for each row of Table 5) and then average across pairs. To compute the *"within index variance"* we first compute the variance between pairs of lines for each index (*i.e.* the variance within Column (1), (3) and (4) of Table 5) and then average across indices.

[30] If we include P^1 the between index standard deviation is almost twice as large as the within index standard deviation (0.115 and 0.064 respectively).

O_0, and this is true for the entire period (see Figure A.3 in the Appendix).[31] Importantly, this underestimation[32] is not merely the result of O_0 being insensitive to the depth of poverty (*i.e.* the gap with respect to the poverty threshold). Indeed, we can alternatively compare our estimates with that obtained using O_1, *i.e.* the poverty-gap ratio below the upper-contour of the absolute and relative lines, a standard gap-sensitive measure. Interestingly, we observe that, except for pair 6 whose absolute threshold is larger, O_1 also finds less poverty reduction than P^0.

We can better quantify to which extent O_0 underestimates the decline in poverty. In order to do so, in Figure 8a we plot the (negative) growth rate of P^0 relative to the (negative) growth rate of O_0 over 3-year periods for the entire sample.[33] We plot each ratio at the end of each 3-year period. For instance, the dot in 1993 represents the growth rate of P^0 over 1990-1993 relative to the growth rate of O_0 over the same period. As the ratio of growth rates between P^0 and O_0 depends on the pair of poverty lines, we focus on our main pair of lines (pair 1) and we briefly comment on the results for the alternative pairs. We observe that the ratio of growth rates is always larger than 1 and tends to increase over time, reaching more than 2 by the end of the period. Given that both P^0 and O_0 decrease in each 3-year period, this means that P^0 decreases systematically more than O_0. More precisely, the rate of decline in global poverty by P^0 is between 8% and 106% larger than that by O_0 (see Table A.8 in the Appendix). We find similar patterns for the alternative pairs of lines (see Figure A.4 in the Appendix). For all alternative pairs, the ratio of growth rates is mostly above 1 and increasing over time. In the last period, P^0 decreases 30% more than O_0 for the most conservative pair of lines and at least 50% more for the remaining pairs.

These numbers show that the underestimation of poverty is economically relevant. Moreover, they are a lower bound on the underestimation as any other index in our family yields an even larger rate of poverty reduction relative to O_0. Specifically, the other extreme of our family, P^1, yields a rate of poverty reduction between 1.3 and 3.2 times larger than O_0 (see Table A.8 in the Appendix).[34]

The key reason that O_0 finds less poverty reduction than P^0 is that O_0 violates our normative assumption. O_0 implicitly considers that all poor individuals are equally poor, regardless of whether they are absolutely poor or only relatively poor. Growth reduces O_0 when a poor individual exits poverty, but it does not record any progress when an

[31] It is worth noting that P^0 is not necessarily always our most conservative measure in terms of poverty reduction. That is, P^1 may decrease more than P^0 in another context or period.

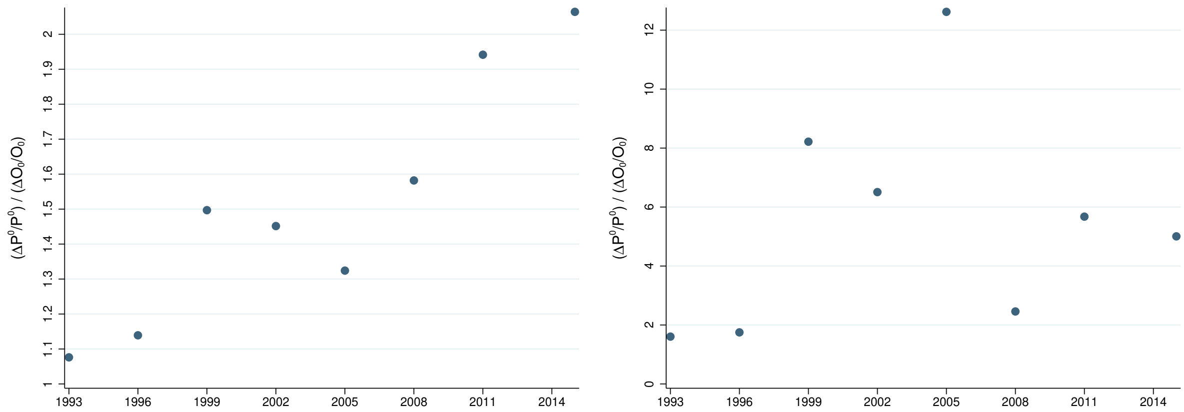
[32] For simplicity, we use the term "underestimation" to refer to the lower poverty reduction found by alternative poverty measures. Of course, only the readers who agree with our normative assumption will consider that alternative measures underestimate poverty reduction.

[33] We consider the following reference years: 1990, 1993, 1996, 1999, 2002, 2005, 2008, 2011 and 2015. As 2014 is not a reference year, the last point in the graph is computed over a 4-year period.

[34] If we compare P^λ against O_1 instead, we observe that P^0 decreases more than O_1 from 1999-2002 onward and by 2011-2015 P^0 decreases almost twice as much as O_0. In turn, P^1 decreases more than O_1 in every period and by the last period P^1 decreases almost three times as much as O_1 (see Table A.8 in the Appendix).

Figure 8: Ratio of growth rates: P^0 over O_0. 3-year periods. Pair of lines 1.

(a) All countries **(b)** Countries with $z_a < z_r$



Notes: These graphs plot the ratio of growth rates between P^0 and O_0 over 3-year periods. Figure (a) includes all countries in the sample. Both P^0 and O_0 decrease in all periods for this sample. Hence, a ratio above (below) 1 implies that P^0 decreases more (less) than O_0. Figure (b) restricts the sample to countries with $z_a < z_r$. For this restricted sample, both P^0 and O_0 decrease in all periods except for 1990-1993, when they both increase. Hence, in 1993 a ratio above (below) 1 implies that P^0 increases more (less) than O_0.

absolutely poor individual crosses the absolute threshold and becomes only relatively poor. In contrast, the hierarchical measures P^λ do record such progress.

As we mentioned, the underestimation of the decline in poverty increases over time. This is because our normative assumption only plays a role in countries for which the relative threshold is larger than the absolute threshold, and the share of countries for which this is the case increases over time. Specifically, O_0 and P^0 (as well as A_0) take the same value in low-income countries where no individual is only relatively poor (when $z_a > z_r$), as revealed by Equations 3 and 4 with $\alpha = 0$ and $\lambda = 0$ respectively.[35] Indeed, absolutely poor individuals all contribute one both to O_0 and P^0. Thus, our normative assumption does not play a role in such countries. O_0 and P^0 register the same progress with growth until these countries grow sufficiently for relative poverty to matter. Now, as soon as the relative threshold becomes larger than the absolute threshold ($z_a < z_r$), some poor individuals exit absolute poverty and become only relatively poor. Then, our assumption kicks in and P^0 takes a smaller value than O_0. The reason is that, if individuals who are only relatively poor contribute one to O_0, they contribute less than one to P^0. Therefore, P^0 records more progress than O_0 when evaluating growth.

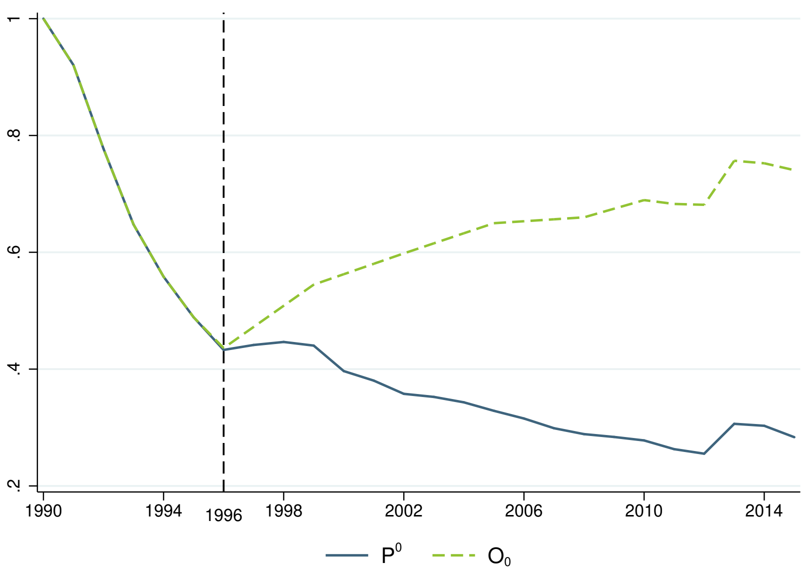
In general, O_α tends to find less poverty reduction than P^λ because the former violates our assumption. This is easily understood when z_r is strongly relative. In that case, any equi-proportionate growth in a country with $z_a < z_r$ leaves O_α unchanged. This behavior of O_α is debatable as such growth typically allows some part of the population to escape absolute poverty. In contrast, this growth reduces P^λ because this measure implicitly considers that being only relatively poor is a form of poverty that is less severe. The

[35] Also note that the IPMs of O_α and P^λ are the same when $z_a > z_r$.

same point is more subtly made when z_r is weakly relative and the growth is not equi-proportionate, even if it remains valid. Our assumption implies less steep iso-poverty curves for P^λ than for O_α (see Figure 2). Therefore, if a given growth process moves the bundle of a poor individual onto a higher iso-poverty curve of O_α (which implies less poverty), then it also moves her bundle onto a higher iso-poverty curve of P^λ. However, the converse is not true. A growth process that lifts the bundle of an absolutely poor individual above z_a, which automatically puts it on a higher iso-poverty curve of P^λ (which implies less poverty), could simultaneously put her bundle on a lower iso-poverty curve of O_α (which implies more poverty).

We illustrate the differential effect of economic growth on P^0 and O_0 with the case of urban China. Figure 9 displays the evolution of poverty in urban China by P^0 and O_0 using the first pair of lines. For the period 1990-1996, urban China has a low income standard and we have $z_a > z_r$.[36] Therefore, both O_0 and P^0 register the same progress over 1990-1996 (a reduction by almost 60%). After 1996, as the income standard is larger and we have $z_a < z_r$, our assumption kicks in and the two measures start diverging. The unequal growth taking place in urban China after 1996 increases O_0 while it reduces P^0 (which registers progress as more and more individuals cross the absolute threshold). Hence, after 1996, the progress in poverty reduction according to P^0 is much larger than that recorded by O_0. Note that we observe a larger reduction in P^0 compared to O_0 for all alternative pairs of lines (see Figure A.5 in the Appendix). In each case, as soon as $z_a < z_r$, P^0 and O_0 diverge and P^0 decreases more.

Figure 9: Evolution of poverty by P^0 and O_0 relative to 1990 for urban China. Pair of lines 1.



Notes: The graph includes all years. The vertical line indicates in which year z_r surpasses z_a.

[36] In 1996, $z_a = z_r$.

This mechanism has clear implications for our analysis. The estimations presented so far (based on Figure 8a) include all countries in the sample for every reference year, even those for which $z_a > z_r$. For the first pair of lines, the share of world's population living in a country with $z_a > z_r$ amounts to 57% in 1990 and 24% in 2015 (see Table A.7 in the Appendix). This increase in the share of population living in a country with $z_a < z_r$ explains why the underestimation of poverty reduction increases over time. To account for this, in Figure 8b we compute the growth rate of P^0 relative to O_0 excluding those countries with $z_a > z_r$ in each 3-year period. Note that the sample obtained consists of different countries in each 3-year period. The underestimation of poverty reduction for this moving sample is striking. The decrease in P^0 is for the most part more than twice as large as the decrease in O_0, and it gets up to twelve times larger. For this restricted sample, P^0 also finds considerably more poverty reduction than O_1 in each period.[37] This conveys an important message for the evaluation of poverty reduction in the future. When most countries have $z_a < z_r$, we can expect that using O_0 will underestimate the rate of poverty reduction by a large extent.

6 Concluding remarks

The approaches of Atkinson and Bourguignon (2001) and Ravallion and Chen (2011) provide two different reasons to evaluate the evolution of income poverty using overall measures, which combine the absolute and relative aspects of income poverty (Ravallion, 2020). Decerf (2017) proposes a normative assumption whose adoption leads to a refinement of the standard overall poverty measures. We discuss the meaning of this assumption under these two approaches and provide some survey evidence that this assumption may collect broad support. More importantly, we show that this assumption has important theoretical and empirical implications for overall poverty comparisons. On the one hand, overall measures that satisfy this assumption may provide comparisons that are independent of the priority assigned to absolutely poor individuals, even when absolute measures disagree with relative measures. On the other hand, these measures record significantly more overall poverty reduction than mainstream measures over 1990-2015 because many absolutely poor individuals became only relatively poor over that period. Mainstream overall measures record slower progress because they need not record an improvement when absolutely poor individuals become only relatively poor as their country grow. Altogether, provided one endorses this assumption, our findings confirm and strengthen positive evaluations of the success achieved against income poverty over the period covered by the Millennium Development Goals.

[37] The only exception is 1990-1993 when both P^0 and O_1 increase. As expected, the underestimation of poverty is even larger if we instead compare P^1 with either O_0 or O_1 for the restricted sample (see Table A.8 in the Appendix).

We acknowledge that the results of our online survey provide limited evidence on the support that our normative assumption may gather. First, the scope of the survey was rather small as only three countries were surveyed and the number of observations per country was limited. Second, online surveys are but one method to evaluate this support. It would be interesting to investigate whether lab experiments or revealed preference methods yield similar results to those of our survey.

Our theoretical results provide a ready-to-use method able to assess the trend of overall income poverty independently of the value selected for the priority parameter. This method can be readily applied in different contexts where both basic subsistence needs and social participation needs are deemed relevant.

References

Anand, S. and Segal, P. (2008). What do we know about global income inequality? *Journal of Economic Literature*, 46(1):57–94.

Atkinson, A. and Bourguignon, F. (2001). Poverty and Inclusion from a World Perspective. In Stiglitz, J. and Muet, P.-A., editors, *Governance, Equity and Global Markets*. Oxford University Press, New York.

Atkinson, A. B. (1987). On the measurement of poverty. *Econometrica*, 2:749–764.

Bourguignon, F. (2015). *The Globalization of Inequality*. Princeton University Press.

Chen, S. and Ravallion, M. (2013). More Relatively-Poor People in a Less Absolutely-Poor World. *Review of Income and Wealth*, 59(1):1–28.

Clark, A. E. and Oswald, A. J. (1996). Satisfaction and Comparison Income. *Journal of Public Economics*, 61:359–381.

Corazzini, L., Esposito, L., and Majorano, F. (2011). Exploring the Absolutist vs Relativist Perception of Poverty using a Cross-Country Questionnaire Survey. *Journal of Economic Psychology*, 32(2):273–283.

Decerf, B. (2017). Why not consider that being absolutely poor is worse than being only relatively poor? *Journal of Public Economics*, 152:79–92.

Decerf, B. (2021). Combining absolute and relative poverty: income poverty measurement with two poverty lines. *Social Choice and Welfare*, 56(2):325–362.

Ferreira, F. H. G., Chen, S., Dabalen, A., Dikhanov, Y., Hamadeh, N., Jolliffe, D., Narayan, A., Prydz, E. B., Revenga, A., Sangraula, P., Serajuddin, U., and Yoshida, N. (2016). A global count of the extreme poor in 2012: Data issues, methodology and initial results. *The Journal of Economic Inequality*, 14(2):141–172.

Foster, J., Greer, J., and Thorbecke, E. (1984). A Class of Decomposable Poverty Measures. *Econometrica*, 52(3):761–766.

Foster, J. and Shorrocks, A. (1991). Subgroup Consistent Poverty Indices. *Econometrica*, 59(3):687–709.

Jolliffe, D. and Prydz, E. B. (2016). Estimating international poverty lines from comparable national thresholds. *The Journal of Economic Inequality*, 14(2):185–198.

Jolliffe, D. and Prydz, E. B. (2017). *Societal Poverty: A Relative and Relevant Measure*. Policy Research Working Papers. The World Bank.

Luttmer, E. F. (2005). Neighbors as Negatices : Relative Earnings and Well-Being. *The Quarterly Journal of Economics*, 120(3):963–1002.

Milanovic, B. (2016). *Global Inequality: A New Approach for the Age of Globalization*. Harvard University Press.

Perez-Truglia, R. (2020). The Effects of Income Transparency on Well-Being : Evidence from a Natural Experiment. *American Economic Review*, 110(4):1019–1054.

Ravallion, M. (2008). On the Welfarist Rationale for Relative Poverty Lines. Policy Research Working Paper 4486.

Ravallion, M. (2014). Income inequality in the developing world. *Science*, 344(6186):851–855.

Ravallion, M. (2020). On measuring global poverty. *Annual Review of Economics*, 12:167–188.

Ravallion, M. and Chen, S. (2011). Weakly relative poverty. *Review of Economics and Statistics*, 93(4):1251–1261.

Ravallion, M. and Chen, S. (2017). Welfare-Consistent Global Poverty Measures. Working Paper 23739, National Bureau of Economic Research.

Ravallion, M. and Lokshin, M. (2010). Who cares about relative deprivation? *Journal of Economic Behavior and Organization*, 2(73):171–185.

Townsend, P. (1979). *Poverty in the United Kingdom*. Penguin Books, Harmondsworth, Middlesex.

World Bank (2015). *A Measured Approach to Ending Poverty and Boosting Shared Prosperity: Concepts, Data, and the Twin Goals*. World Bank, Washington, DC.

World Bank (2018). Poverty and shared prosperity 2018: Piecing together the poverty puzzle. Technical report, Washington, DC.

A Appendix

A.1 Hierarchical indices with non-linear contributions

Measure P^λ is the linear case of the more general family P^λ_α, for which individual contributions are defined as

$$\left(1 - d^\lambda(y_i, \overline{y})\right)^\alpha,$$

where $\alpha \geq 0$ is the poverty aversion parameter. Measure P^λ is obtained when assuming $\alpha = 1$. As a result, all individuals who have the same poverty status (being absolutely poor or being only relatively poor) have the same priority. One may wonder how would our results be affected when allowing priority to differ within a given poverty status, *i.e.* when taking $\alpha \neq 1$.

Providing a definitive answer to this question may seem out of reach because, when $\alpha \neq 1$, there are no straightforward conditions as the ones stated in Proposition 2. However, there are intuitive reasons for the strong reduction in overall poverty documented in this paper to be robust to alternative values of α. First, taking $\alpha > 1$ increases the priority of individuals at the bottom of income distributions, typically the absolutely poor individuals, while it decreases the priority of individuals close to the poverty threshold, typically the only relatively poor individuals. Given that we observe a large reduction in absolute poverty over 1990-2015, we expect that taking $\alpha > 1$ would lead to an even larger overall poverty reduction than the one found when taking $\alpha = 1$. Second, taking $\alpha < 1$ would have the opposite effect, and this should lead to a smaller overall poverty reduction. Yet, when taking the smallest value $\alpha = 0$, our indices are all equivalent to the head-count ratio, *i.e.* equivalent to O_0, and as shown in the next section, this index finds an overall poverty reduction in the world between 37% and 57%. Such reduction is smaller than the one found by P^λ, but it is very different from the slight increase found by the purely relative measure R_1. Altogether, we should not expect that changing the value of α could overturn our result. There are no values for the pair (λ, α) such that the evolution of P^λ_α would become arbitrarily close to that of R_α.

A.2 P^λ comparisons may depend on λ when A_α and R_α disagree

In general, when there is a disagreement between absolute measures and relative measures, the overall poverty comparison with P^λ depends on the value taken by parameter λ. This is for instance illustrated in Table A.1. The absolute and relative poverty lines are defined as in the previous example. Distributions x' and y' have the same value of mean income and therefore share the same relative poverty threshold. Both distributions feature three individuals: individual 1 is absolutely poor, individual 2 is only relatively poor and individual 3 is non-poor. Individual 1 earns \$0.5 more in y' than in x', but individual 2 earns \$1 less in y' than in x'. Thus, when moving from distribution x' to distribution y', the gain of the absolutely poor is smaller than the loss of the relatively poor. The absolute measure A_1 and the relative measure R_1 disagree on x' and y'. The overall poverty comparison with P^λ of x' and y' depends on the priority assigned to absolutely poor individuals. When $\lambda = 0.3$, the priority given to the absolutely poor is low and P^λ is larger in y' than in x'. When $\lambda = 0.7$, the priority given to the absolutely poor is high and P^λ is smaller in y' than in x'. The overall poverty comparison of x' and y' depends on the value chosen for parameter λ. In our terminology, their overall poverty comparison is *ambiguous*.

Table A.1: Overall poverty comparison of x' and y' depend on λ.

	$i=1$	$i=2$	$i=3$	z_a	z_r	A_1	R_1	$P^{0.3}$	$P^{0.7}$
Distribution x'	1	4	25	1.9	5	0.16	0.33	0.36	0.24
Distribution y'	1.5	3	25.5	1.9	5	0.07	0.37	0.41	0.21

Notes: We set $z_a = 1.9$ and $z_r(\overline{y}) = 0.5\overline{y}$ where $\overline{y}$ is mean income.

In Table A.2, we provide two distributions x^* and y^* for which the absolute measure A_0 and the relative measure R_0 disagree, but the overall poverty comparison with P^λ of x^* and y^* depends on the priority λ assigned to absolutely poor individuals.

Table A.2: Overall poverty comparison of x^* and y^* depend on λ.

	$i=1$	$i=2$	$i=3$	z_a	z_r	A_0	R_0	P^0	P^1
Distribution x^*	1.7	2.5	10.8	1.9	2.5	0.33	0.33	0.33	0.035
Distribution y^*	1.9	2.3	10.8	1.9	2.5	0	0.66	0.44	0

Notes: We set $z_a = 1.9$ and $z_r(\overline{y}) = 0.5\overline{y}$ where $\overline{y}$ is mean income.

A.3 Proof of Proposition 1

Consider distributions x'' and y'' given in Table A.3.

Table A.3: Unambiguous comparison of x'' and y''

	$i=1$	$i=2$	z_a	z_r	A_α	R_α
Distribution x''	1.7	4.3	1.9	1.5	>0	0
Distribution y''	2.3	7.7	1.9	2.5	0	>0

Notes: We set $z_a = 1.9$ and $z_r(\overline{y}) = 0.5\overline{y}$ where $\overline{y}$ is mean income.

First, we show that $A_\alpha(x) > A_\alpha(y)$ for all $\alpha \geq 0$. Individual 1 is absolutely poor in x, which implies that $A_\alpha(x) > 0$ for all $\alpha \geq 0$. There is no absolutely poor in y, which implies that $A_\alpha(y) = 0$ for all $\alpha \geq 0$.

Second, we show that $R_\alpha(x) < R_\alpha(y)$ for all $\alpha \geq 0$. There is no relatively poor in x, which implies that $R_\alpha(x) = 0$ for all $\alpha \geq 0$. Individual 1 is relatively poor in y, which implies that $R_\alpha(y) > 0$ for all $\alpha \geq 0$.

Third, we show that $P^\lambda(x) > P^\lambda(y)$ for all $\lambda \in [0, 1]$. This follows from Eq. (5) because individual is absolutely poor in x whereas she is only relatively poor in y.

A.4 Proof of Proposition 2

Take any two distributions $x, y \in Y$.

First, we show that P^λ is *linear* in λ for any distribution $y \in Y$. That is, $P^\lambda = B + \lambda C$, where B and C do not depend on λ. P^λ adds the contributions of absolutely poor individuals P^a_λ to the contributions of only relatively poor individuals P^r_λ:

$$P_\lambda(y) = \underbrace{\frac{1}{n}\sum_{i=1}^{\mathbf{q_a(y)}} 1 - d^\lambda(y_i, \overline{y})}_{:=\mathbf{P^a_\lambda(y)}} + \underbrace{\frac{1}{n}\sum_{i=\mathbf{q_a(y)+1}}^{\mathbf{q(y)}} 1 - d^\lambda(y_i, \overline{y})}_{:=\mathbf{P^r_\lambda(y)}}. \tag{A.6}$$

Developing these two terms, we get

$$P^a_\lambda(y) = \frac{q_a(y)}{n} - \lambda\frac{q_a(y)}{n}\bar{Y}^a(y).$$

where $\bar{Y}^a(y) = \frac{\hat{y}^a}{z_a}$ and $\hat{y}^a = \sum_{i=1}^{q_a(y)} \frac{y_i}{q_a(y)}$ and

$$P^r_\lambda(y) = \frac{q(y) - q_a(y)}{n}\left(1 - \bar{Y}^r(y)\right) - \lambda\frac{q(y) - q_a(y)}{n}\left(1 - \bar{Y}^r(y)\right)$$

where $\bar{Y}^r(y) = (\hat{y}^r - z_a)/(z_r(\overline{y}) - z_a)$ and $\hat{y}^r = \sum_{i=q_a(y)+1}^{q(y)} \frac{y_i}{q(y)-q_a(y)}$.

Together, we get:

$$P^\lambda(y) = \frac{q_a(y)}{n} + \frac{q(y) - q_a(y)}{n}\left(1 - \bar{Y}^r(y)\right) - \lambda\left[\frac{q_a(y)}{n}\bar{Y}^a(y) + \frac{q(y) - q_a(y)}{n}\left(1 - \bar{Y}^r(y)\right)\right], \tag{A.7}$$

which proves that P^λ is linear in λ.

Second, we show that $\frac{P^0(x)}{P^0(y)} \leq \frac{P^1(x)}{P^1(y)}$ implies $\frac{P^0(x)}{P^0(y)} \leq \frac{P^\lambda(x)}{P^\lambda(y)} \leq \frac{P^1(x)}{P^1(y)}$ for all $\lambda \in [0, 1]$. As P^λ is linear, we can write $P^\lambda(x) = B + \lambda C$ and $P^\lambda(y) = D + \lambda E$. Inequality $\frac{P^0(x)}{P^0(y)} \leq \frac{P^1(x)}{P^1(y)}$ can be rewritten as $BE \leq CD$. Take any $\lambda \in [0, 1]$, we cannot have $\frac{P^0(x)}{P^0(y)} > \frac{P^\lambda(x)}{P^\lambda(y)}$ because this inequality is equivalent to $BE > CD$. In turn, we cannot have $\frac{P^\lambda(x)}{P^\lambda(y)} > \frac{P^1(x)}{P^1(y)}$ because this inequality is also equivalent to $BE > CD$.

Finally, using the same reasoning, we also have that $\frac{P^0(x)}{P^0(y)} \geq \frac{P^1(x)}{P^1(y)}$ implies $\frac{P^0(x)}{P^0(y)} \geq \frac{P^\lambda(x)}{P^\lambda(y)} \geq \frac{P^1(x)}{P^1(y)}$ for all $\lambda \in [0, 1]$, which concludes the proof.

A.5 Tables and figures

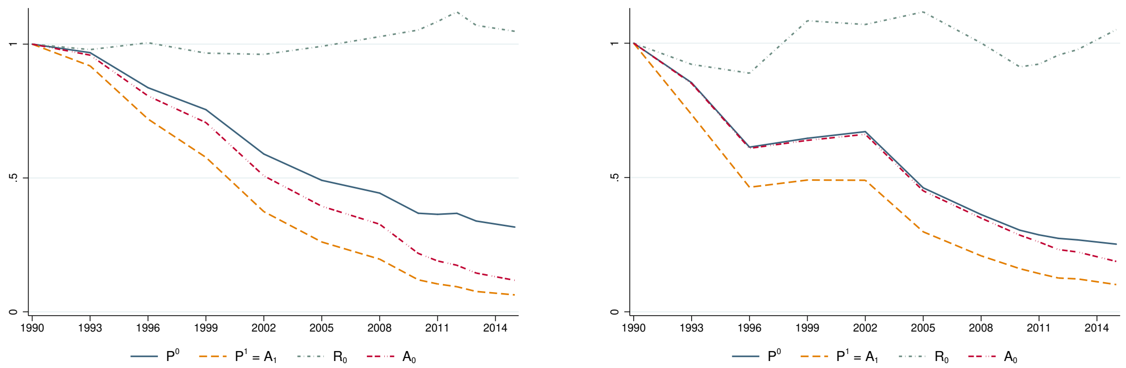
Table A.4: Statistics and global poverty excluding China and/or India. Values of 2015 relative to 1990. Pair of lines 1.

Countries excluded	Mean inc. (1)	P^0 (2)	$P^1 = A_1$ (3)	A_0 (4)	R_0 (5)	R_1 (6)	Dis. (7)	Unam. (8)
Both China & India	1.23	0.64	0.47	0.53	0.94	0.86	No	Yes
Only China	1.24	0.55	0.38	0.44	0.97	0.89	No	Yes
Only India	1.44	0.42	0.24	0.27	1.06	1	No	Yes

Note: Variables are defined as in Table 4.

Figure A.1: Evolution of poverty for Asia excluding China and India relative to 1990. Pair of lines 1.

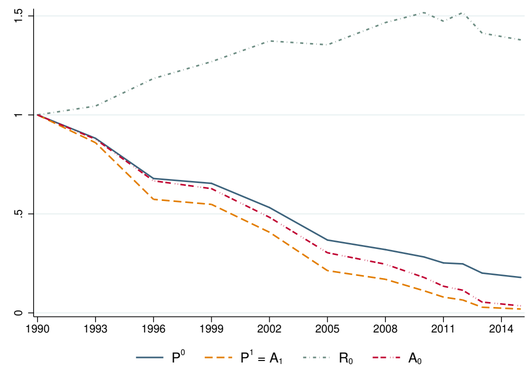
(a) East Asia & Pacific excluding China **(b)** South Asia excluding India



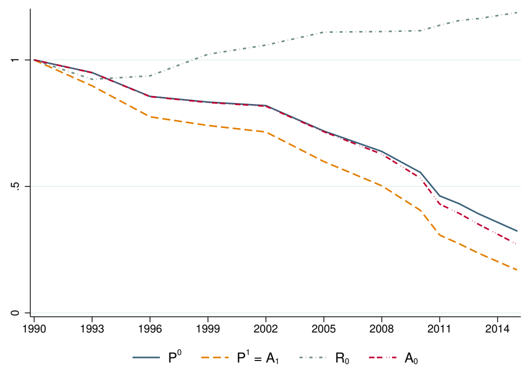
Notes: These graphs plot the evolution of poverty in East Asia and South Asia excluding China and India respectively. They display poverty as measured by different indices for all reference years until 2015 relative to 1990. They include all countries in each region, except from China and India, with available information in any reference year.

Figure A.2: Evolution of poverty in East and South Asia, and Sub-Saharan Africa relative to 1990. Pair of lines 1.

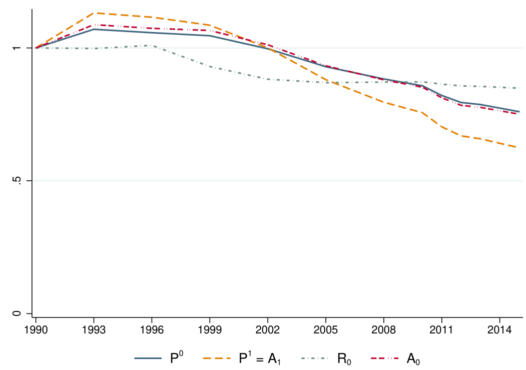
(a) East Asia and Pacific



(b) South Asia



(c) Sub-Saharan Africa



Notes: These graphs plot the evolution of poverty in each region as measured by different indices for all reference years until 2015 relative to 1990.

Table A.5: Disagreement status between A_0 and R_0, and A_1 and R_1. 2015 vs. 1990. Pair of lines 1.

Disagreement between absolute and relative measures	A_0 vs. R_0		A_1 vs. R_1	
	Freq. (1)	% (2)	Freq. (3)	% (4)
No	109	71	99	65
Yes	44	29	54	35
Total	153	100	153	100

Notes: Odd columns display the frequency of agreements and disagreements between A_0 and R_0 (Column 1), and A_1 and R_1 (Column 3) when comparing 2015 to 1990. Even columns show the corresponding percentages.

Table A.6: Change in P^0 by ambiguity status. 2015 vs. 1990. Pair of lines 1.

Direction of change in P^0 by ambiguity status	Freq.	%
Ambiguous		
P^0 increases	14	9
P^0 decreases (less than halved)	9	6
Unambiguous		
P^0 increases	31	19
P^0 decreases (less than halved)	72	44
P^0 decreases (at least halved)	37	23
Total	163	100

Notes: The first column displays the frequency of countries where P^0 increases or decreases (by more or less than half) when comparing 2015 to 1990. These cases are divided by the ambiguity status, that is whether the evolution of P^0 is independent of λ (unambiguous) or not (ambiguous). The second column shows the corresponding percentages.

Table A.7: Share of world's population with $z_a < z_r$. Pair of lines 1.

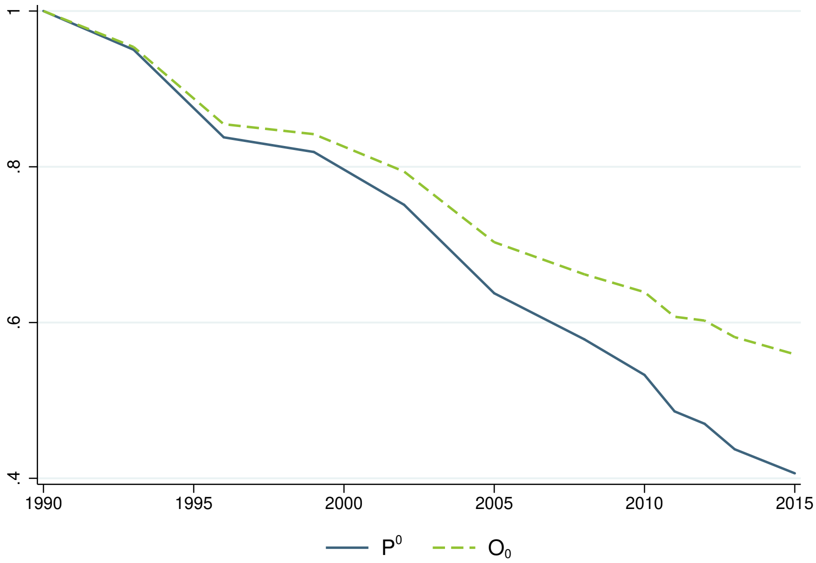
1990	1993	1996	1999	2002	2005	2008	2011	2015
43	41	48	48	49	52	71	74	76

Table A.8: Ratio of growth rates: P^λ over O_α. 3-year periods. Pair of lines 1.

3-year period	P^0/O_0		P^1/O_0		P^0/O_1		P^1/O_1	
	All (1)	$z_a < z_r$ (2)	All (3)	$z_a < z_r$ (4)	All (5)	$z_a < z_r$ (6)	All (7)	$z_a < z_r$ (8)
1990-1993	1.08	1.61	1.34	4.73	0.95	1.40	1.18	4.13
1993-1996	1.14	1.75	1.71	3.07	0.88	5.51	1.32	9.68
1996-1999	1.50	8.22	1.82	16.11	0.90	1.08	1.09	2.12
1999-2002	1.45	6.51	2.33	21.27	1.02	3.45	1.64	11.28
2002-2005	1.32	12.62	2.03	43.33	1.05	15.59	1.60	53.52
2005-2008	1.58	2.46	2.32	4.65	1.19	1.91	1.74	3.61
2008-2011	1.94	5.67	3.15	15.90	1.58	3.38	2.57	9.47
2011-2015	2.06	5.01	3.17	14.43	1.92	4.65	2.94	13.39

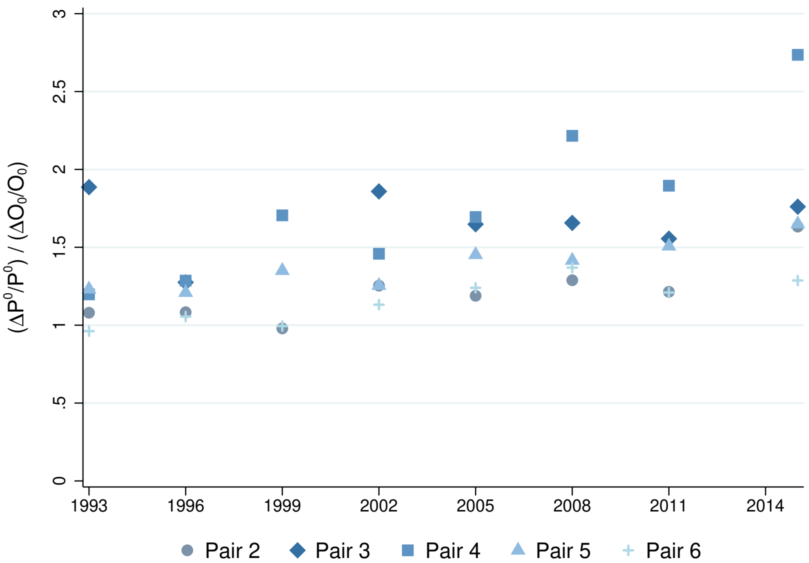
Notes: This table displays the ratio of growth rates between either P^0 or P^1 and O_0 or O_1 over 3-year periods. Odd columns include all countries in the sample. The four indices decrease in all periods for this sample. Hence, a ratio above (below) 1 implies that P^0 or P^1 decrease more (less) than O_0 or O_1. Even columns restrict the sample to countries with $z_a < z_r$. For this restricted sample, all indices decrease in all periods except for 1990-1993, when they all increase. Hence, in 1990-1993 a ratio above (below) 1 implies that P^0 or P^1 increase more (less) than O_0 or O_1.

Figure A.3: Evolution of global poverty by P^0 and O_0 relative to 1990. Pair of lines 1.



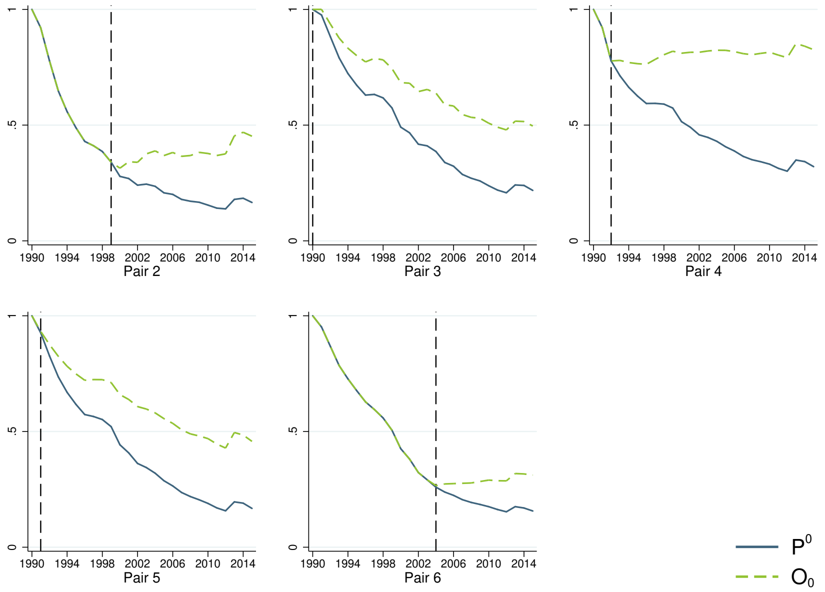
Notes: The graph plots the evolution of global poverty by P^0 and O_0 for all reference years until 2015 relative to 1990.

Figure A.4: Ratio of growth rates: P^0 over O_0. 3-year periods. Alternative pairs of lines.



Notes: This graph plots the ratio of growth rates between P^0 and O_0 over 3-year periods for all alternative pairs of lines (2 to 6). Only observations with ratios < 3 are displayed. This excludes one observation for the pair of lines 3, which takes value 5 in 1999. For all pairs of lines, both P^0 and O_0 decrease in all periods. Hence, a ratio above (below) 1 implies that P^0 or P^1 decrease more (less) than O_0 or O_1.

Figure A.5: Evolution of poverty by P^0 and O_0 relative to 1990 for urban China. Alternative pairs of lines.



Notes: These graphs include all years. The vertical lines indicate in which year z_r surpasses z_a for each alternative pair of lines.